MODELLING THE WATERJET CABLE TRENCHING PROCESS ON SAND DUNES
Numerous offshore wind farms have been recently installed in the southern part of the North Sea. Their infield and export cables are buried for protection against dropped or dragged objects. In sandy soils, burial is carried out by remotely operated tracked vehicles. Two swords with waterjets are used to fluidise the sand and generate a backward flow of the water-sediment mixture. The area's highly variable seabed topography, characterised by sand waves and mega-ripples, can influence the trenching process. At the moment, it is not possible to make an accurate estimate of the influence of sand dunes on the trenching process.

The trench formation process is split into two parts: a front section where the seabed is eroded by waterjets (erosion model) and a rear section where the sand grains are settling in a backward flow (sedimentation model).

The erosion model is made based on the assumption that the specific energy required to fluidise sand is equal to the specific energy required to cut sand with a blade. The blade is considered to have a small blade angle and to operate at zero meter water depth, following Miedema (2015). For a given jetting configuration and trench dimensions, this results in a limiting trencher velocity. A volume balance between situ soil, waterjet flow and entrained flow gives the backwash flow rate and concentration. The last two are used as input for the sedimentation model.

The sedimentation model relates water flow, sediment transport, bed evolution and trench width evolution, based on the shallow water equations. The governing equations represent horizontal momentum and mass conservation of the water-sediment mixture and horizontal mass conservation of the sediment. A numerical one-dimensional finite volume model is proposed which is solved on a staggered grid.

An elastic cantilever beam model is used to determine the cable shape as it sinks in the trench. Subsequently, the depth of lowering of the cable is determined by the intersection of the cable and trench shape. The combined fluidisation, sedimentation and cable model is validated against full-scale field data.

**Introduction**

Offshore cables are commonly buried for protection against dropped or dragged objects. In sandy soils, burial is carried out by remotely operated tracked vehicles, see Figure 1A. Two swords with waterjets are located in...
between the tracks of the vehicle, on either side of the cable. Water is pumped through the swords to fluidise the sand and generate a backward flow of the water-sediment mixture, see Figure 1B. The cable located on the seabed is lowered into the water-sediment mixture due to its own weight. Due to waves, tides and currents, sandy seabeds are often not flat but can contain seabed features such as sand waves and mega-ripples. When burying a cable in these seabed features, the achieved depth of lowering shows an oscillating profile with the maximum depth of lowering achieved at the peaks of these features and minimum at their troughs. Models to predict the achieved depth of lowering are currently not able to account for the influence of seabed features, therefore an attempt has been made in modelling the effect of seabed features on the depth of lowering.

To make an accurate prediction of the depth of lowering of a cable, modelling is divided into three parts: a model for the erosion section, a model for the sedimentation section and a model for the cable deflection. The erosion model determines the maximum trencher velocity at which the seabed can still be eroded. Furthermore, it provides flow input values for the sedimentation model. The sedimentation model determines the shape of the trench behind the vehicle. By iteratively calculating the cable shape in the cable deflection model, the point of intersection with the trench shape is determined. It is assumed that cable remains fixed in this point of intersection, hereby giving the achieved depth of lowering. See Figures 2 and 7A for an illustration of the jet trenching model working principle.

**Erosion model**

At the front of the trench, sand is eroded by the water flow from the jet swords. It is assumed that the erosion process results in well-mixed backward flow of water and sediment. Via a volume balance the backward flow rate and sediment concentration is determined. Via a specific energy approach the maximum trencher velocity is determined at which the jets are still able to sufficiently erode the seabed. It is assumed that below this trencher velocity there are no problems to erode the seabed.

**Limiting trencher velocity using a specific energy approach**

Miedema (2015) derived a theory for the in-situ production of jets in a draghead. It is based on the assumption that the specific energy required to fluidise sand is equal to the specific energy required to cut sand with a blade, having a small blade angle and at zero-metre water depth. The in-situ production \( Q_{\text{situ}} \) (sand plus pore water) of the jet trencher is given by equation (1), where \( d_{\text{max}} \) and \( h_0 \) are initial trench depth and width respectively and \( v_t \) is the trencher velocity. The in-situ production can also be defined by dividing jet power \( P_j \) by specific energy \( E_{sp} \).

\[
Q_{\text{situ}} = A_{\text{trench}} \cdot v_t = d_{\text{max}} \cdot b_0 \cdot v_t = \frac{P_j}{E_{sp}}
\]

(1)

The specific energy is determined by assuming it to be equal to that of non-cavitating cutting, given by equation (2). Where \( \varepsilon \) is dilatancy, \( k_m \) is mean permeability and \( \rho_w \) is seawater density. Horizontal force coefficient \( c_1 \) must be calibrated using experiments. Miedema (2015) suggests a value of 0.12. This value is based on calibration with experiments done by Combinatie Speurwerk Baggertechniek published by Jong (1988).

\[
E_{sp} = c_1 \cdot \rho_w \cdot g \cdot h_0 \cdot v_t \frac{\varepsilon}{k_m}
\]

(2)

For cutting sand with a blade the parameter \( h_0 \) is the layer thickness. Since the jet swords have numerous jets spaced vertically, this does not directly relate. A reasonable assumption is to consider the trench as a whole and therefore assume \( h_0 \) to be equal to the trench depth \( d_{\text{max}} \). The ratio of mean permeability to dilatancy can be approximated using the Kozeny Carman equation, resulting in the following relation.

\[
k_m \approx 10 \cdot k_0
\]

(3)

Total jet power \( P_j \) is given by the product of jet pressure and flow rate, where flow rate is determined by jet pressure \( p_j \) and nozzle diameter \( D_j \). By combining the equations mentioned before, the maximum trench depth \( d_{\text{max}} \) is found as a function of soil parameters, jetting parameters and trencher speed \( v_t \). Since the sword depth is an input, the maximum trencher velocity can be determined by the intersection of sword depth and maximum trench depth \( d_{\text{max}} \).

\[
d_{\text{max}} = \frac{p_j \cdot \eta_j \cdot \sqrt{\frac{2p_j}{\rho_w}} \cdot \frac{\pi}{4} (a_j \cdot D_j)^2 \cdot 10 \cdot k_0}{c_1 \cdot v_t^2 \cdot \rho_w \cdot g \cdot b_0} \left[ \frac{1}{2} \right]
\]

(4)

**FIGURE 2**

Schematisation of the interaction between erosion, sedimentation and cable model with corresponding outputs.
over-depth is dependent on trencher velocity, jetting power and seabed permeability. After calibration a reasonable assumption was found to be \( a_{OD} = 0.03 \).

\[
h_{OD} = a_{OD} \cdot d_{\text{max}}
\]

(7)

The initial trench width \( h_t \) is assumed to be approximately the same as the sword separation distance (outside to outside), plus a small margin. The in-situ flow rate is now given by equation (8).

\[
Q_{\text{situ}} = v_t \cdot A_{\text{trench}} = v_t \cdot (h_{\text{sword}} + h_{OD}) \cdot b_0
\]

(B)

**Entrainment of ambient water**

A certain amount of ambient water will entrain the flow before reaching the transition between erosion and sedimentation. Due to the complex flow pattern and lack of experimental data it is difficult to say which mechanisms are taking place. Therefore, to estimate the entrainment flow rate some simplifications and assumptions have to be made.

- Flow entrainment is based on entrainment calculations for free non-cavitating jets.
- Water is only entrained at the backside of the jet (see Figure 5A). On the front side of the flow idealisation of the erosion section, volume balance of in-situ, jet and entrained flow rate.
the jets, in-situ soil is loosened which is not considered as entrainment but is included in the volume conservation.
• Water is entrained in an individual jet over a distance equal to the vertical spacing between individual jets. \( \Delta h_{\text{jet}} \) (see Figure 5).
• Water is only entrained in the forward jets located on the top half of the jet swords, the inward and backwash jets are neglected.

The total entrained flow rate is determined by the summation of entrainment per individual jet. For a single jet sword the number of forward jets at the top half of the sword is given by \( N \). Consequently, the total entrained flow rate is given by equation (9).

\[
Q_E = 2 \cdot \sum_{i=1}^{N} Q_{E,i}
\]  

(9)

Of which the entrainment for a single jet, for a distance from zero to \( \Delta h_{\text{jet}} \), is given by equation 10.

\[
Q_{E,i} = \alpha_E \cdot \pi \int_0^{\Delta h_{\text{jet}}} r_u(s) \cdot u_u(s) \, ds
\]  

(10)

In a free turbulent jet, two regions can be defined: a region of flow development and a region of fully developed flow. For the flow development region, the entrainment coefficient is half of the fully developed flow region entrainment coefficient. For a free non-cavitating jet, a reasonable assumption is \( \alpha_E = 0.085 \) in the fully developed flow region and a value of \( k = 77 \) for the empirical constant \( k \) as given in Nobel (2013). Also the development of the uniform flow velocity \( u_u \) and jet radius \( r_u \) is different in the flow development region and region of fully developed flow. Analytical expressions are derived in Lee and Chu (2003) and are given by equations (11), (12), (13) and (14).

\[
\text{Fors} < s_{0r}: \quad u_u(s) = u_0 \left( \frac{1}{\sqrt{2k}} \cdot \frac{k}{s + \frac{1}{2}D_j} \right)
\]  

(11, 12)

\[
\text{Fors} \geq s_{0r}: \quad u_u(s) = \frac{1}{2} \cdot \frac{k}{\sqrt{2k}} \cdot \frac{k}{s}
\]  

(13, 14)

The position \( s_{0r} \) where the transition from flow development region to developed flow region is, can be determined by \( s_{0r} = k/2 = 6.2D_j \). Now, equations (10), (11), (12), (13) and (14) can be combined and simplified resulting in equation (15). The first term gives the entrainment in the flow development zone and the second term is the entrainment in the developed flow region up to a distance \( \Delta h_{\text{jet}} \).

\[
Q_{E,i} = \pi \alpha_E \cdot \frac{6.2D_j}{2} \int_0^{\Delta h_{\text{jet}}} \frac{k}{s + \frac{1}{2}D_j} \left( u_0 \cdot \frac{D_j \sqrt{k/2}}{s + \frac{1}{2}D_j} \right) \, ds + \frac{\pi}{2} \alpha_E \cdot u_0 \cdot D_j \left( \Delta h_{\text{jet}} - 6.2 \cdot D_j \right)
\]  

(15)

Output to sedimentation model

The flow rate and concentration are determined by equations (16) and (17). Where the entrained flow rate is corrected by a factor \( (1-c_o) \). This correction results from the assumption that the entrained water is not completely sediment-free but has a concentration equal to the output concentration. Equation (16) is therefore implicit and must be solved iterative.

\[
c_o = \frac{1 - n_0 \cdot Q_s}{Q_s + Q_j + Q_E \cdot (1 - c_o)}
\]  

(16)

\[
Q_s = Q_s + Q_j + Q_E \cdot (1 - c_o)
\]  

(17)

Sedimentation model

The sedimentation model describes the backward flow containing water and suspended sediment. Input variables used from the erosion model are initial trench dimensions, flow rate and sediment concentration. Included in the sedimentation model is breaching of trench sidewalls, entrainment of ambient water and erosion/sedimentation of the trench bottom. Breaching is included via the active wall velocity, \( v_{act} \), in Figure 7B. The sidewalls are assumed to be flat and remain vertical. Furthermore, it is assumed that the material coming from the side walls is mixed instantaneous in the backward flow, hereby conserving the rectangular trench shape. All main parameters are illustrated in Figures 7A and B. Notable are the constant concentration \( c \) and velocity \( u \) over the vertical axis, and the distinction between initial seabed porosity \( n_0 \) and re-settled seabed porosity \( n_r \).

Shallow water equations for flow in a rectangular channel with variable cross-section

To model the flow of water and sediment behind the trencher, the so-called Shallow Water Equations (SWE) are used. Since they were first proposed by Saint Venant (1871), these equations are also referred to as the Saint-Venant equations. It is assumed that the flow is one-dimensional, hereby reducing the
system of equations to only three.

- continuity of the total fluid volume (water plus sediment), see equation (18)
- continuity of sediment volume, see equation (19)
- conservation of momentum, see equation (20)

Furthermore, it is assumed that all quantities are uniform over the cross-section and vertical velocities are neglected. To simplify the solving of the momentum equation, it is rewritten so that the concentration is not present in the time derivative anymore, similar to He et al. (2014) and Cao et al. (2004).

\[
\frac{\partial (hb)}{\partial t} + \frac{\partial (hbu)}{\partial x} = (v_E - v_{sed}) \cdot b + 2 \cdot v_{wall} \cdot d_{tr} 
\]

(18)

\[
\frac{\partial (hbc)}{\partial t} + \frac{\partial (hbcu)}{\partial x} = -v_{sed}(1 - n_c)b + 2 \cdot v_{wall}(1 - n_o)d_{tr} 
\]

(19)

\[
\frac{\partial (hbu)}{\partial t} + \frac{\partial (hbu^2)}{\partial x} + \frac{1}{2} g \frac{\partial (h^2)}{\partial x} = S_{bed} + S_f + S_{sed} + S_c + S_w 
\]

(20)

Where \( h \) is flow height, \( b \) is trench width, \( u \) is mean flow velocity and \( c \) is sediment concentration. Furthermore, \( v_E \) is the entrainment velocity, \( v_{sed} \) is the sedimentation velocity (i.e. vertical velocity of the bed), \( v_{wall} \) is the active wall velocity due to breaching, \( n_c \) and \( n_o \) are the porosity of the initial and re-settled seabed respectively and \( d_t \) the trench depth. The source terms on the right hand side in the momentum equation in equation (20) are given separately in equations (21), (22), (23), (24) and (25) for improved readability. These source terms \( S_{bed} \), \( S_f \), \( S_{sed} \), \( S_c \) and \( S_w \) account for the bed gradient, bed friction, sediment exchange, concentration gradient and divergence of the trench width respectively. The source term \( S_c \) is not present in the classical form of the shallow water equations where the width is considered constant. However, it arises in the derivation of the shallow water equations in a channel of varying width, see for example Robert and Wilson (2011) or Siviglia et al. (2008).

\[
S_{bed} = -g \cdot h \cdot b \left( \frac{\partial x}{\partial x} - \tan(\theta) \right) 
\]

(21)

\[
S_f = -u^2(2 \cdot d_{tr} + b) 
\]

(22)

\[
S_{sed} = v_{sed} \cdot u \cdot b \left( \frac{\rho_{settled} - \rho}{\rho} \right) 
\]

(23)

\[
S_c = \frac{1}{2} g \cdot b \cdot h^2 \frac{\partial c}{\partial x} 
\]

(24)

\[
S_w = \frac{1}{2} g \cdot h \cdot \frac{\partial b}{\partial x} 
\]

(25)

The system of equations is completed with the description of the evolution of trench width \( b \) in time, see equation (26), and evolution of bed elevation \( z \) in time, see equation (27).

\[
\frac{\partial b}{\partial t} = 2 \cdot v_{wall} 
\]

(26)

\[
\frac{\partial x}{\partial t} = v_{sed} 
\]

(27)

Lastly, the definition of mixture density \( \rho \) is given by equation (28) and the density of the seabed after it has re-settled again by equation (29).

\[
\rho = c \cdot \rho_s + (1 - c) \cdot \rho_w 
\]

(28)

![FIGURE 6](image)

Output values to the sedimentation model in case of a supercritical flow (\( u_t \) and \( h_t \)) or subcritical flow (\( Q_j \)).

![FIGURE 7](image)

Trench side view (A) and cross-section (B), showing all main parameters used in the sedimentation model (\( t_0 < t_1 < t_2 \)).
\[ \rho_{sett} = n_s \cdot \rho_w + (1 - n_s) \cdot \rho_s \]  

(29)

One-dimensional finite volume scheme on a staggered grid

The equations are solved on a one-dimensional staggered grid, using a finite volume scheme. In the staggered grid, the flow height \( h \) and concentration \( c \) are discretised at the center of each cell. The flow velocity \( u \) is discretised at the interfaces between the cells. Corresponding to the discretised variables, the control volume for continuity equations is centered around the \( h, c \) and \( u \) variables, whereas the control volume of the momentum equation is centered around velocity \( u \) and is thus staggered with respect to the continuity control volume (see Figure 8). The change in width is only a function of the wall velocity which is a known constant, see equation (26). Therefore, the width is known at every grid point and every time step and thus no approximations are required. The continuity equation for the total volume and the continuity equation for the sediment volume are discretised explicit in time and upwind for the fluxes, see equations (30) and (31).

The variables that are not defined on the grid are denoted with a hat and determined via an upwind approximation. Time steps are indexed by superscript \( n \) and space steps by subscript \( i \).

The momentum equation is discretised on a staggered grid, see equation (32) for the discretised equation.

With the discretised source terms given by equations (33), (34), (35), (36) and (37). The evolution of trench width in time and the bed evolution in time is given in discretised version by equations (38) and (39) respectively.

\[ \frac{b_{i+1}^{n+1} - b_i^n}{\Delta t} = 2 \cdot v_{wall} \]  

(38)

\[ \frac{x_i^{n+1} - x_i^n}{\Delta t} = (v_{sed})_i^n \]  

(39)

To prevent the trench width from diverging unbounded, the width evolution is stopped when the trench depth has decreased up to a...
certain threshold value. This threshold value is set at 0.05 metres. Furthermore, the mixture density at the cell faces is given by equation (40).
\[ \hat{\rho}^n_{i+1/2} = \rho_s \hat{\epsilon}^n_{i+1/2} + \rho_w (1 - \hat{\epsilon}^n_{i+1/2}) \]
(40)

As stated before, the variables not defined on the grid are denoted with a hat and are determined via an upwind approximation (see equations 41, 42 and 43).

\[ \hat{h}^n_{i+1/2} = \begin{cases} h^n_{i+1} & \text{if } u^n_{i+1/2} \geq 0 \\ h^n_{i+1} & \text{if } u^n_{i+1/2} < 0 \end{cases} \]
(41)
\[ \hat{\epsilon}^n_{i+1/2} = \begin{cases} \epsilon^n_{i+1} & \text{if } u^n_{i+1/2} \geq 0 \\ \epsilon^n_{i+1} & \text{if } u^n_{i+1/2} < 0 \end{cases} \]
(42)
\[ \hat{n}^n_{i+1/2} = \begin{cases} n^n_{i+1/2} & \text{if } \left( \frac{1}{2} \left( u^n_{i+1/2} - u^n_{i+1/2} \right) \right) \geq 0 \\ u^n_{i+1/2} & \text{if } \left( \frac{1}{2} \left( u^n_{i+1/2} - u^n_{i+1/2} \right) \right) < 0 \end{cases} \]
(43)

The sedimentation velocity \( \nu_{sed} \), entrainment velocity \( \nu_e \) and friction velocity \( \nu_f \) are defined as a function of \( h, u \) and \( \epsilon \) on either the cell centers or faces. The explicit upwind approximations are used when a variable that is not defined on the grid is required.

Treatment of moving boundary cell

The trentcher is moving to the left in a fixed grid, thus being located in different grid cells in time. Due to stability issues, it is not possible for the trentcher boundary to move exactly one grid cell every time step. It will take several time steps for the trentcher to move one grid cell. As a result, the boundary cell – denoted by subscript \( b \) – will grow in size with velocity \( \nu_f \) for the continuity control volumes – see equation (44) – and with velocity \( \nu_{v2} \) for the momentum control volume. See Figure 9, where \( \Delta x^* \) is the old boundary cell size and \( \Delta x^{n+1} \) is the size of the boundary cell in the new time step. The flux going through the left boundary should therefore be corrected with the growth velocity of the boundary cell.

\[ \Delta x^{n+1}_b = \Delta x^n_b + \nu_f \cdot \Delta t \]
(44)

To give the treatment of the boundary cell in a readable expression, the contiuity equation for shallow water flow with constant width is used in equation (45). This is a simplified version of equation (18), with all the source terms included in variable \( S \).

\[ \frac{\partial h}{\partial t} + \frac{\partial \hat{u}}{\partial x} = S \]
(45)

For an explicit treatment of the source terms and the fluxes, the discretised expression of equation (45) is now given by equation (46), which can be solved for \( h^{n+1} \).

\[ h^{n+1}_b \Delta x^{n+1}_b - h^n_b \Delta x^n_b = \frac{1}{\Delta t} \left( S^n_b \cdot \Delta x^n_b - \left[ (h u)^{n+1/2} - (h (u + \nu_f))^2 \right]^{n-1/2} \right) \]
(46)

Empirical equations for model closure

The last unknowns in the set of equations are the active wall velocity \( \nu_{wall} \), sedimentation velocity \( \nu_{sed} \), entrainment velocity \( \nu_e \) and friction velocity \( \nu_f \). The wall velocity for a vertical wall is given by equation (47) [Rhee, 2015], where \( k_o \) is initial permeability, given by equation (48) by Adel (1987). \( \phi \) is the internal friction angle, \( \Delta = (\rho_s - \rho_w) / \rho_w \) is kinematic viscosity, \( n_s \) initial porosity and \( D_{15} \) 15th percentile grain size.

\[ \nu_{wall} = -10 \cdot k_o \cdot \Delta \cdot \frac{\sin(\phi - \pi/2)}{\sin \phi} = -10 \cdot k_o \cdot \Delta \cdot \cot(\phi) \]
(47)
\[ k_o = \frac{g}{1600 \cdot D_{15}^2} \frac{n_s^2}{(1 - n_s)^2} \]
(48)

The sedimentation velocity is given by equation (49), where \( S \) and \( E \) are sedimentation and erosion flux respectively and \( \rho_s \) is sediment density. For a detailed explanation of the sedimentation and erosion fluxes, see Rhee (2010).

\[ \nu_{sed} = \frac{S - E}{\rho_s \cdot (1 - n_s - \epsilon)} \]
(49)

The entrainment velocity is included via equation (50) where entrainment coefficient \( a_e \) is determined via an empirical function in equation (51) proposed by Parker et al. (1987). The function converges to 0.075 for non-stratified flow. The Richardson number is defined as \( Ri = g \cdot h u^2 / \nu^2 \), with reduced gravity \( g = g(\rho_s - \rho_e) / \rho_s \) where \( \rho_s \) is mixture density and \( \rho_e \) water density.

\[ \nu_e = a_e \cdot u \]
(50)
\[ a_e = \frac{0.075}{(1 + 718 \cdot R_i^{1/4})^{1/2}} \]
(51)

The friction velocity is included to account for the friction of the flow at the bed and sidewalls of the trench. The friction velocity is given by equation 52, where \( f \) is an empirical factor and \( u \) the layer averaged flow velocity. A bed friction factor of \( f = 0.024 \) is used, within the range mentioned in Garcia (1990).

\[ u_c = \sqrt{\frac{f}{8}} \cdot u \]
(52)

Boundary and initial conditions

On the left (moving) boundary, the flow height, width, concentration and velocity are imposed. In the case of a subcritical flow it is sufficient to specify only the flow rate instead of both the height, width and velocity. However, the volume flux through the moving boundary is \( h \cdot (u + \nu_f) \), which cannot be replaced by a flow rate \( q \) due to the presence of \( \nu_f \). Therefore, flow velocity \( \nu_c \) is determined manually as an input, and can be tweaked to make sure the simulation is stable.

The right boundary is an outflow boundary, where a zero-gradient boundary condition is imposed. For this assumption to be valid, fluctuations should be minimal towards the right boundary. The initial conditions are therefore chosen such that the bed elevation and trench width are constant close to the right boundary. To achieve this, the initial location of the moving boundary is set at a certain distance (default 5 metres) from the right boundary. The bed elevation is set at zero and the flow velocity, height and width are set equal to the output values of the erosion model. The bed elevation and trench width are then constrained such that they will remain constant in this section.

Stability of the scheme

For the scheme to be stable it has to satisfy the Courant-Friedrichs-Lewy (CFL) condition, given by equation S3. Where CFL is the
dimensionless CFL number, \( u_{\text{max}} \), is the maximum velocity in the domain, \( \Delta t \) the time step and \( \Delta x_{\text{min}} \), the smallest possible grid size in the domain.

\[
CFL = \frac{|u_{\text{max}}| \cdot \Delta t}{\Delta x_{\text{min}}} \leq 1
\]

The trencher boundary moves to the left in the grid in time, dependent on trencher velocity and time step. In one-time step, the boundary cell will grow by a distance equal to \( \Delta x_{\text{min}} \), which is also the smallest grid size occurring in time. The CFL condition for this cell is given by equation S4. From this relation it can be concluded that the CFL condition for the boundary cell can only be fulfilled if the trencher velocity is greater or equal to the flow velocity \( |v| \geq |u_{\text{max}}| \). In other words, the stability of the scheme, with regard to the CFL condition for the boundary cell is independent of the time step. However, since a quite robust scheme (explicit, upwind) is used, the CFL number can be higher than one. Also, since there is only one grid cell that does not fulfill the CFL condition, the error created dampens out in the rest of the domain.

\[
CFL = \frac{|u_{\text{max}}| \cdot \Delta t}{v \cdot \Delta x} = \frac{|u_{\text{max}}|}{v}
\]

Verification
This section aims to verify whether the proposed numerical scheme is able to capture the phenomena that can occur in shallow water flows. Examples of these phenomena are propagating shocks and transitions between subcritical and supercritical flows and vice versa (hydraulic jumps). To do this verification, the numerical scheme is applied on several scenarios for which exact analytical solutions are known. Analytical solutions are taken from the SWASHES library (Delestre et al., 2016), in which numerous analytical solutions for the shallow water equations are summarised. Four different cases are used for this verification: a dam-break, subcritical flow over a bump and transcritical flow over a bump with and without a hydraulic jump. Results are given in Figures 10, 11, 12 and 13.

Results of the verification show that the scheme is shock-capturing and performs reasonably well. Some deviation in the discharge is observed at the hydraulic jump location in Figure 13. However, this is not an issue within the current application of the scheme since an overall trench profile is the objective and not accurate local values of the discharge.
Cable deflection model

The burial depth of the cable is determined by the intersection of the cable shape and the re-settled seabed. The cable shape is based on an elastic, hyperstatic cantilever beam model which is uniformly loaded. The left hand side is completely fixed and the right hand side is restrained in rotation. The residual lay tension in the cable is applied at the right hand side of the cantilever beam. It is assumed that the position of the cable remains fixed when the touchdown point intersects with the trench shape. The analytical solution for the cantilever beam is given in Vanden Berghe et al. (2011). Exact assumptions in the derivation of the equation are unknown, therefore a verification has been done by comparing the analytical solution to numerical solutions generated by OrcaFlex, which is a dynamic analysis package used within the offshore industry.

FIGURE 12
Comparison of analytical and numerical solution of the staggered upwind scheme for a transcritical flow over a bump, without hydraulic jump. Shown is flow elevation (A) and discharge (B).

FIGURE 13
Comparison of analytical and numerical solution of the staggered upwind scheme for a transcritical flow over a bump, with hydraulic jump. Shown is flow elevation (A) and discharge (B).

FIGURE 14
Model results for $D_{15} = 0.15$mm, $D_{50} = 0.3$mm and $v_{t} = 0.1$m/s. Plots show a side view of the trench (A), a top view of the trench with the development of trench width (B) and the depth averaged sediment concentration (C).
\[ z(x) = -\frac{ql}{EI} \sqrt{\frac{EI}{T}} \cdot \cosh \left( \frac{T}{EI} \right) \cdot \cosh \left( \frac{T}{EI} \right) \]

\[ - \tanh \left( \frac{T}{EI} \right) \cdot \sinh \left( \frac{T}{EI} \right) - 1 \cdot \frac{q}{2T} x^2 - \frac{qL^2}{2T} \]

(55)

Where \( z \) is the cable deflection measured from the seabed, \( q \) is cable weight, \( T \) is residual lay tension, \( L \) is distance until touchdown point, \( EI \) is bending stiffness and \( x \) is the distance from the start of the trench. The cable deflection equation is incorporated in the sedimentation model and is solved each time a new grid cell is created at the moving boundary. The cable is not lowered in clear water but in a mixture of sediment and water, the cable weight should thus be corrected. A single reference concentration is chosen to correct the cable weight. This reference concentration is chosen to be equal to the concentration at the interface of erosion and sedimentation model.

\[ q_{\text{sturry}} = q_{\text{water}} + c_{\text{ref}} \cdot D^2 \cdot \frac{\pi}{4} (\rho_w - \rho_s) \cdot g \]

(56)

Where \( q_{\text{sturry}} \) is the cable weight in the water sediment mixture, \( q_{\text{water}} \) is the cable weight submerged in water, \( c_{\text{ref}} \) the reference concentration of sediment, \( D \) is cable diameter, \( \rho_w \) and \( \rho_s \) are water and sediment density respectively.

**Results**

A typical output of the model is given in Figure 14, showing a side view, top view and the concentration development behind the trencher.

**Model validation**

To validate the jet trenching model, averages of the depth of lowering are taken per cable section (monopile to monopile). To account for uncertainties in grain size and residual cable tension, a minimum and maximum depth of lowering case is considered. The minimum case is based on \( (d_{\text{c}} = 0.2 \text{mm}, d_{\text{w}} = 0.4 \text{mm}, T = 5 \text{kN}) \) and the maximum case on \( (d_{\text{c}} = 0.1 \text{mm}, d_{\text{w}} = 0.25 \text{mm}, T = 2 \text{kN}) \). Per individual cable section the average jet pressure and trencher velocity is extracted from logs recorded during trenching. Furthermore, the sword depth and measured and predicted depth of lowering.
Cable depth of lowering for a range of trencher velocities and four different $d_{50}$.

Cable depth of lowering for a range of sand dune lengths and constant sand dune height of 0.4 metres.

The effect of having a higher trencher velocity depends on grain sizes. In coarse sand, the increase in depth of lowering is larger than in fine sand for the same increase in trencher velocity (see Figure 18). Therefore, it is more important to have a high trencher velocity in coarse sand than in fine sand.

Sand dunes, characterised by their height and length, are modelled as a simple sinusoidal profile. Due to the seabed profile, also the depth of lowering shows an oscillating profile. To indicate this behaviour, the bandwidth (minimum and maximum values) and mean depth of lowering is plotted. To investigate sensitivity of sand dune length, the height is kept constant and only dune length is varied (see Figure 19). The bandwidth shows a clear local minimum at a wavelength of approximately 7.5 metres. This is an interesting wavelength since it is approximately equal to the layback of the cable (distance from start of trench to touchdown point of cable). For a wavelength of half the layback and 1.5 times the layback, there is a maximum in the bandwidth. Thus when the start of the trench is in phase with the touchdown point of the cable the variation of depth of lowering is minimum. However, the mean depth of lowering is hardly influenced.

A similar plot but now for sand dune height is given in Figure 20. The bandwidth shows to increase almost linearly with sand dune height, and again the mean depth of lowering is hardly influenced by sand dune height.

**Discussion**

The amplitude at which the depth of lowering oscillates when buried in sand dunes, is shown to be underestimated by the model compared to field data. An explanation could be the relatively simple cable equation used in the model. This equation takes the tension as a constant and uses this to calculate the cable shape. However, it is expected that while the cable is being lowered in the trench on sand dunes, the tension in the

### FIGURE 18
Cable depth of lowering for a range of trencher velocities and four different $d_{50}$.

### FIGURE 19
Cable depth of lowering for a range of sand dune lengths and constant sand dune height of 0.4 metres.
Numerous offshore wind farms have been recently installed in the southern part of the North Sea. Their infield and export cables are buried for protection against dropped or dragged objects. In sandy soils, burial is carried out by remotely operated tracked vehicles. Two swords with waterjets are used to fluidise the sand and generate a backward flow of the water-sediment mixture. The southern part of the North Sea has a highly variable seabed topography characterised by sand waves and mega-ripples. These seabed features can significantly influence the trenching process. At the moment, it is not possible to make an accurate estimate of the influence of sand dunes on the trenching process.

The trench formation process is split into two parts; a front section where the seabed is eroded by waterjets (erosion model) and a rear section where the sand grains are settling in a backward flow (sedimentation model). Both models as well as an elastic cantilever beam model – to determine the cable shape as it sinks in the trench – are delineated in this article. The combined fluidisation, sedimentation and cable model is validated against full-scale field data.

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In coarse sand, the increase in depth of lowering is larger than in fine sand for the same increase in trencher velocity.
By investigating the effect of grain sizes and trencher velocity, it has been shown by the model that it is more important to have a high trencher velocity in coarse sand than in fine sand.