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Modelling the Sedimentation Process in a Trailing Suction Hopper Dredger

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Since 1985 Cees van Rhee has been engaged with research for the dredging industry, the first five years at WL|Delft Hydraulics and from 1990 to the present at Ballast HAM Dredging. At Ballast HAM he was employed at the Research Department, Estimating Department and worked for two years in Hong Kong where he was responsible for the Production and Planning of one of the large land reclamation projects that were executed at that time. From 1997–2001 he was posted for two days per week at Delft University of Technology where a PhD research programme, financed by major Dutch dredging contractors, was executed. This article is part of this research programme.

Abstract

In the last decades and in the near future, large areas of land, especially in the Far East, were and will be reclaimed using trailing suction hopper dredgers. During the overflow phase of the loading stage of the dredging cycle, a part of the dredged volume of sand will not settle in the hopper but will be transported with the overflow discharge overboard. For production, sand quality and environmental reasons it is important to predict the amount of these so-called overflow losses. The total volume of losses as the influence on the loaded (and lost) particle size distribution (PSD) is important.

In 1997 a research programme was started to get more understanding of the sedimentation process onboard a hopper dredger. The goal of the programme was to develop a numerical model that can be used to predict the influence of the relevant parameters as hopper geometry, sand, discharge and concentration on the overflow losses (and which fractions of the PSD will be lost). The research programme consists of three parts: laboratory experiments, development of numerical models and full-scale validation of the models. In this article the developed 1DV numerical model will be presented. The 1DV-model is one-dimensional, but in contrast to most existing models in the vertical direction. The influence of the PSD distribution is modelled using a coupled system of transport equations (convection-diffusion) for the different grain sizes. The numerical results will be compared with experiments.

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Introduction

In the last decades, large areas of land were reclaimed using trailing suction hopper dredgers (TSHD), and in the near future more large reclamation projects will be executed, especially in the Far East. During the overflow phase of the loading stage of the dredging cycle, a part of the dredged volume of sand will not settle in the hopper but will be transported with the overflow discharge overboard. For production, sand quality and environmental reasons it is important to predict the amount of these so-called overflow losses.

A trailing suction hopper dredger (TSHD) is a sea-going vessel that is equipped with one or two suction pipes, which are lowered to the sea bottom during dredging (Figure 1). From the bottom a sand-water mixture is sucked up and discharged into the cargo hold, the so-

Figure 1. Artist's rendering of a trailing suction hopper dredger, with cutaway underwater showing suction pipes and hopper.

called hopper. Sand settles from this mixture and when the hopper is filled with sand the ship sails to the location where the sand is being discharged or pumped ashore. Generally a part of the incoming sand will not settle during loading in the hopper, but will be lost overboard with the overflow mixture. To achieve an accurate prediction of the production and the quality of sand being dredged knowledge of the amount and of the fractions of sand lost is important.

For this reason a research programme was initiated and financed by the Dutch dredging industry. The ultimate goal of this research programme is to gain more insight in the sedimentation process and to develop a model to describe this process. The research programme consists of three parts:
1. Experimental investigation using model tests
2. Numerical modelling using one-dimensional (1DV) and two-dimensional approaches
3. Prototype verification of the numerical model.

In this article the one-dimensional numerical model which was developed is described. Finally, the theory will be compared with the experimental results.

Process Description

During the first phase of the experimental programme large-scale model tests were carried out. A rectangular laboratory flume (dimensions Length x Width x Height = 12 x 3 x 2 m) represented the cargo hold of the ship, the hopper. From the velocity and concentration measurement inside the flume and visual observation through the glass sidewall the following flow pattern could be distinguished (Figure 2). The inflow is located at the left side and the overflow at the right side of the hopper.

The hopper area can be divided into 5 different sections:
1. Inflow section
2. Settled sand or stationary sand bed
3. Density flow over settled bed
4. Horizontal flow at surface towards the overflow
5. Suspension in remaining area

In the inflow section the incoming mixture flows towards the bottom and forms an erosion crater and density current. From this current, sedimentation will take place which leads to a rising sand bed. The part of the incoming sediment which does not settle will move upward into suspension. At the water's surface the vertical supply of water and sediment creates a strong horizontal flow towards the overflow section.

The loading process will be continued until the hopper is completely filled with sand or when the sand losses grow to an uneconomical level.

Apart from the inflow section the flow in the suspended part of the hopper is basically one-

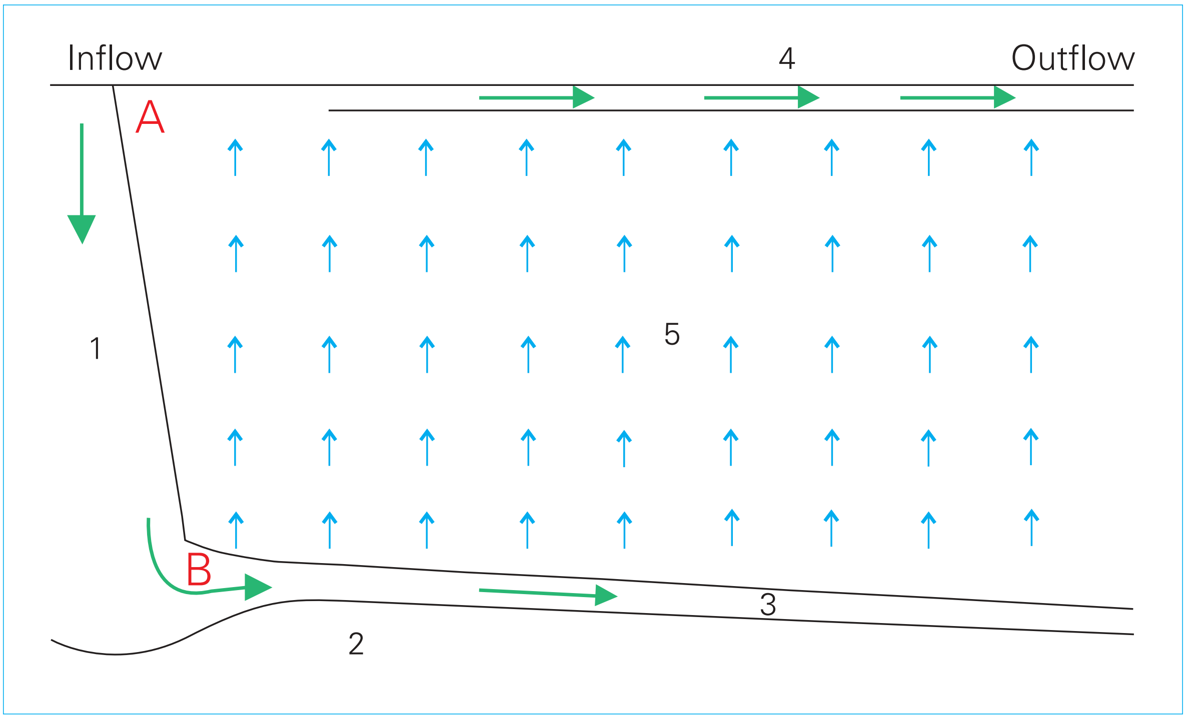


Figure 2. Observed flow field in the hopper.

dimensional. In the horizontal direction the (measured) concentration is very uniform (vertically layered). For more details of the experiments and results reference is made to Van Rhee (2001).

Overflow losses

A very important quantity during the loading process is the overflow loss. Two different definitions of this quantity are used. The loss can be defined as:

– the ratio of the outflow and inflow sand flux at a certain moment; or
– the ratio of the total outflow and inflow (sand) volume.

The overflow flux is defined as:

$$OV_{flux}(t) = \frac{Q_o(t)\,C_o(t)}{Q_i(t)\,C_i(t)} \qquad (1)$$

The (cumulative) overflow loss is defined as:

$$OV_{cum}(t) = \frac{\int_0^t Q_o(t)\,C_o(t)dt}{\int_0^t Q_i(t)\,C_i(t)dt} \qquad (2)$$

in which Q is the discharge and C the volume concentration. The indexes i and o relate to the inflow and outflow respectively.

Overview of Existing 1DV Numerical Modelling

In the past a number of models to describe the hopper sedimentation process have been published. These models were all based on the ideal settlement basin theory of Camp (1946) and Dobbins (1944). In this theory the hopper is divided in three areas: the inflow, outflow and settling section (Figure 3). From the inflow section the mixture flows over the total depth (like in river flow) in the settling section.

Both the flow fields as the diffusion coefficient are based on velocity distribution (uniform or logarithmic) over the total depth. Vlasblom and Miedema (1995) extended this theory with the influence of hindered settling and the influence of the rising sand bed. Owing to the rising sand bed, horizontal velocity increases during loading which leads to increased scour.

The effect on the overflow losses is, however, limited since it is assumed that velocity is uniform over depth and therefor apart from at the very last moment will remain at a low value.

More recently Ooijens (1999) extended this model by adding dynamics to the system. In the Vlasblom and Miedema model the concentration in the hopper is always equal to the inflow concentration and outflow concentration reacts instantaneously on the calculated settling efficiency.

Ooijens adds the time effect by regarding the hopper as an ideal mixing tank. The calculated concentration in the hopper is used for the settling efficiency calculation. The extension is an evident improvement, since it enables, for instance, the influence of overflow level variation on the calculation. The basis of the method is still, however, the Camp theory that is based on a flow field and turbulence distribution, not according to reality, as was shown in the "Process Description".

The 1DV Model – Introduction

It was mentioned in the "Process Description" that apart from the inflow section, near the bottom and near the water surface, the flow in the suspended part of the hopper is basically one-dimensional. In the horizontal direction the (measured) concentration is very uniform (vertically layered). Based on this observed flow field of Figure 2, the flow can be further schematised as indicated in Figure 4. Four different areas can be noticed in this figure. From bottom to top:

1. Settled sand
2. Area where sand is supplied to the model (simplification of the density current)
3. Suspension
4. Overflow section (the overflow section normally present at one or two locations is uniformly distributed over the total surface)

In area 2 the inflowing sand flux is prescribed. A part (depending on grain size and local concentration) will settle into zone 1. The remainder will be transported into zone 3. At the top (area 4) the sand can escape into the overflow. The concentration development in zone 3 is described with the advection-diffusion equations for all fractions. The theory will be outlined in the next paragraph.

Like all models this model simplifies reality:

– (possible) erosion or decreased sedimentation owing to the bottom shear stress from the density current is not taken into account.
– the suspension layer is supplied with a sand-water mixture with a concentration and discharge (point B in Figure 2). It is assumed that these quantities at this location are the same as at the inflow location in the hopper (point A in Figure 2). In reality this is not the case since owing to entrainment between points A and B (see Figure 2) mixing will take place between the inflow section and the suspended section.
– reduction of the effective hopper area owing to the presence of an erosion crater is not taken into account.
– the sand is uniformly distributed over the total surface of the hopper, so in fact an ideal inflow system is created (an infinite number of inflow points equally divided over the total hopper area). In reality the inflow is only located in a finite number of locations (in practice very often only at one location) and horizontal sediment transport must distribute the sand over the hopper surface. This horizontal transport is accompanied by a horizontal mixture velocity which, when above a certain threshold value, can reduce the

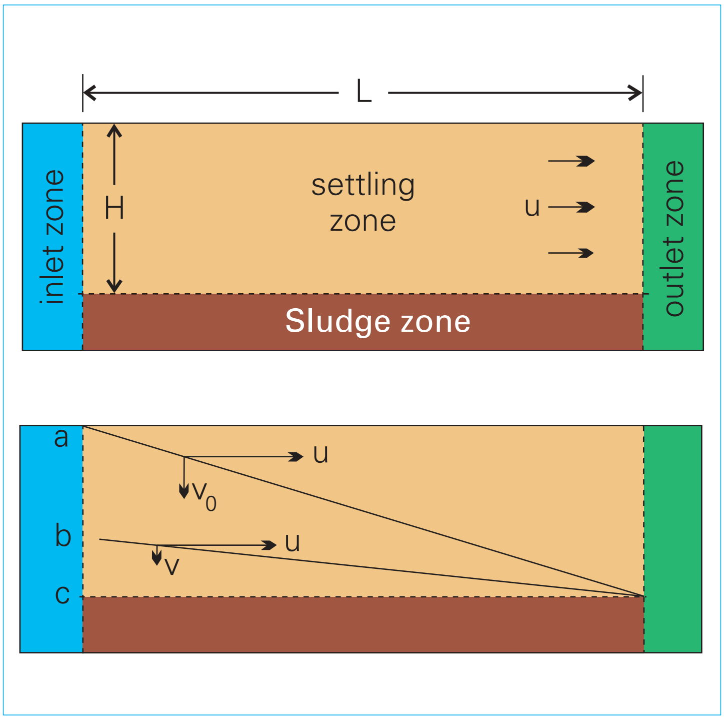
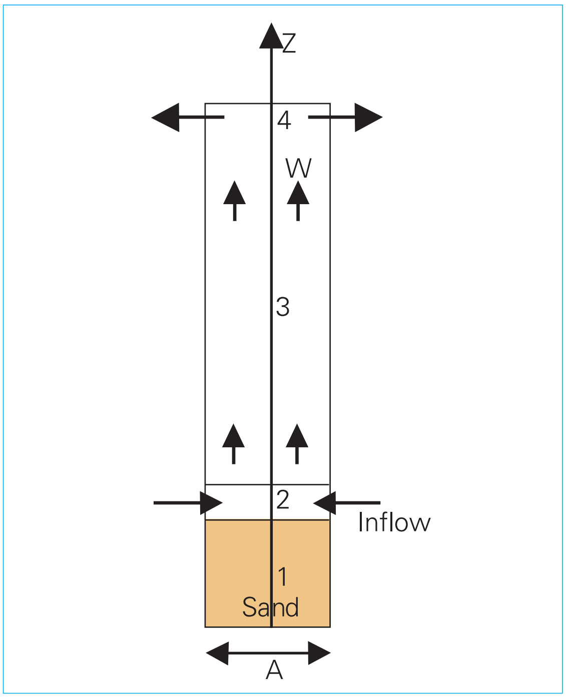


Figure 3. Simplified settling process according to Camp.

Figure 4. Definition 1DV Model.



sedimentation velocity. Owing to scale effects this mechanism will not play an important role with the model hopper sedimentation.

Instead of the horizontal one-dimensional approach of the Camp-like models with a horizontal supply of sand on one side and overflow on the other, this model supplies sand from the bottom (fed by the density current) and the overflow will be located at the top.

It will be shown that this will implement the influence of the hopper load parameter and the mutual interaction of the different grain sizes of the particle size distribution in a relatively simple way. The latter effect is totally absent in the Camp model; every fraction is calculated independently.

Basic Equations of the 1DV Model

The vertical transport of sediment in zone 3 is described with the one-dimensional advection-diffusion equation. Using the grain-size distribution the incoming sand flux can be distributed over the different fractions. The cumulative particle size distribution (PSD) is used in the model to take the different grain sizes into account. If the PSD is presented with N+1 points, N fractions are used.

The advection-diffusion equation for a fraction i can be written as:

$$\frac{\partial c_i}{\partial t} = -\frac{\partial}{\partial z}(c_i v_{z,i}) + \frac{\partial}{\partial z}\left[\varepsilon_z \frac{\partial c_i}{\partial z}\right] + q_{s,i} \tag{3}$$

In this equation c_i is the concentration $v_{z,i}$ and the vertical velocity of a certain fraction. The vertical diffusion coefficient is represented with ε_z. The equation includes a source term $q_{s,i}$ that is used to insert the sediment into the system.

If the equation is solved for a mono-sized suspension (only one grain diameter present), the fall velocity for that grain size can be substituted for v_z. The fall velocity of a grain is a function of the grain properties and the concentration. In general (Richardson and Zaki 1954), this relation is written as the product of a reduction function and the fall velocity of a single grain w_0 as in equation (4).

$$w_s = (1 - c)^n \bullet w_0 \tag{4}$$

The exponent n_i is a function of the Particle Reynolds number defined as $\text{Re}_p = w_o D/\nu$. Richardson and Zaki (1954) provided different relations depending on the value of Re_p. A smooth presentation can be achieved using a logistic curve given with:

$$\frac{A - n}{n - B} = C\,\text{Re}_p^{\alpha} \tag{5}$$

Or, more conveniently:

$$n = \frac{a + b\,\text{Re}_p^{\alpha}}{1 + c\,\text{Re}_p^{\alpha}} \tag{6}$$

For the coefficients, the values are reported as seen in Table I.

The different approaches are compared in Figure 5. It is clear that the expression according to Rowe (1987) is a smooth representation of the original Richardson and Zaki relations. The relation according to Garside and Al-Dibouni (1977) shows the same trend, but provides somewhat higher values for the exponent. Using the values according to Di Felice (1999) very high values for the exponent are found. This relation is however only valid for dilute mixtures.

When a multi-sized mixture is simulated, the situation becomes more complicated since the different fractions will have a mutual influence. The simplest approach, often used in numerical models used to compute suspended sediment transport, is to use the total concentration in the reduction function.
The vertical velocity of a certain fraction is in that case calculated with equation (7). (N is the number of fractions, and n is the exponent valid for that fraction).

$$v_{z,i} = w_{0,i}(1 - \bar{c})^{n_i} \;;\; \bar{c} = \sum_{i=1}^{N} c_i \tag{7}$$

This approach is however not correct because the effect of the return flow of large particles on the small particles is not included. With this simple relation all particles will move in the same direction, while in reality it is possible that small particles move in the opposite direction owing to the return flow of the large particles.

If the effect of the grain size is to be modelled correctly, a more sophisticated approach is needed. A better

Table I. Values of coefficients.

Author(s)	**Re$_p$**	**Conc.**	**a**	**b**	**c**	**∞**
Garside et al. (1977)	$0.001 < \text{Re}_p < 3 \times 10^4$	$0.04 < c < 0.55$	5.1	0.27	0.1	0.9
Rowe (1987)	$0.2 < \text{Re}_p < 10^3$	$0.04 < c < 0.55$	4.7	0.41	0.175	0.75
Di Felice (1999)	$0.01 < \text{Re}_p < 1 \times 10^3$	$0 < c < 0.55$	6.5	0.3	0.1	0.74

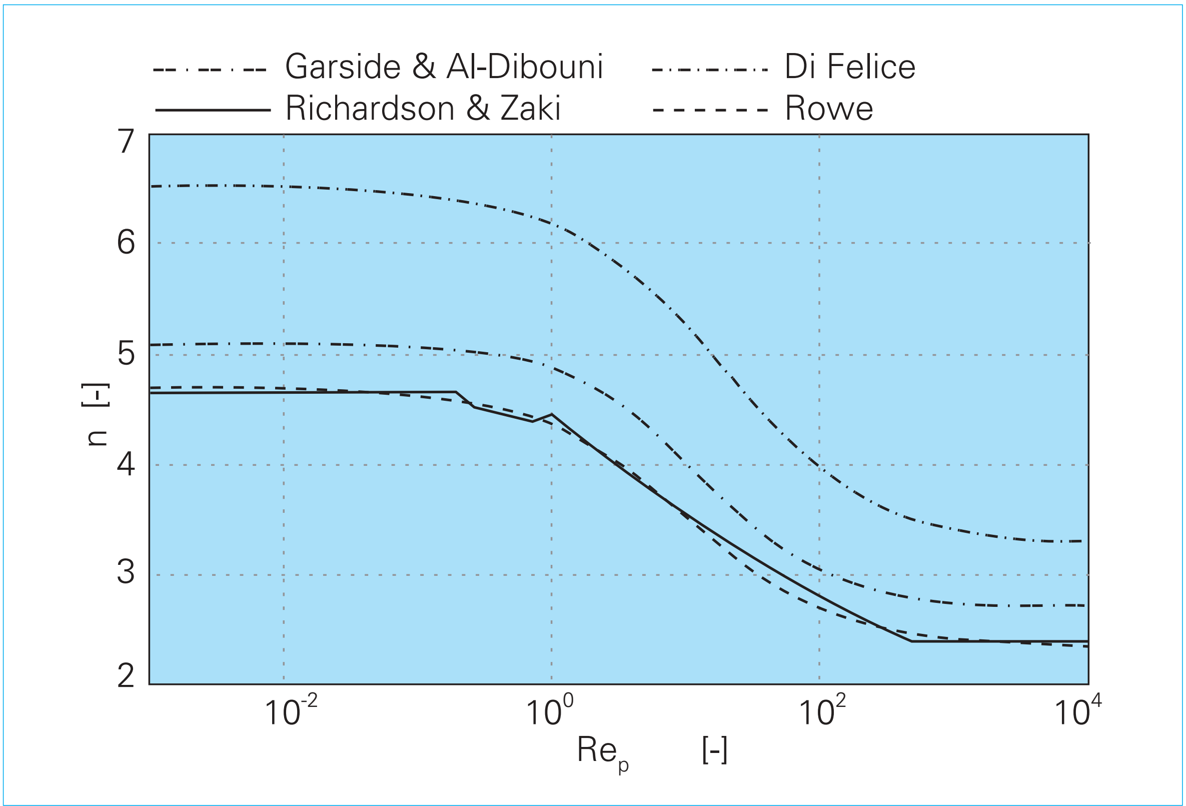


Figure 5. Exponent of hindered settling function.

approach is to assume that every grain settles with a certain slip velocity relative to the fluid velocity (Mirza and Richardson 1979).

$$v_{z,i} = v_w - w_{s,i} \tag{8}$$

The slip velocity is calculated according to Mirza and Richardson (1979) with:

$$w_{s,i} = w_{0,i}\,(1 - \bar{c})^{\,n_i - 1} \tag{9}$$

This results directly from the hindered settling equation, since the settling particles create a return flow that has to flow through an area $(1 - \bar{c})$. In this approach the influence between two or more different fractions is present with the total concentration and the return flow of all particles. The particle-particle interactions between different fractions are, however, not included and as a result this approach does not give good agreement with experimental data for particles with large differences in size (or density).

A relatively simple method to include the interparticle influences for different fractions was proposed by Selim et al (1983). On the fall velocity w_0 of the larger grains, the influence of the smaller grains is taken into account by a correction of the specific density for the larger grains. It is assumed that a grain with a certain size settles in a suspension formed by the grains with a smaller size.

To solve the advection-diffusion equation for all grain sizes the combined action of all grain sizes must be quantified. This can be done using the volume balance in vertical direction for both sand and water:

$$\sum_{i=1}^{N} v_{z,i} c_i + \left[1 - \sum_{i=1}^{N} c_i\right] v_w \quad \frac{Q_{in}}{A} = w \tag{10}$$

Note that the vertical bulk or mixture velocity w is based on the discharge into the hopper and the total hopper area as a result of the simplifications mentioned earlier in "1DV Model – Introduction". Together with equations (8) and (9) this last relation forms a system of N+1 equations with N+1 unknowns (v_w and $v_{z,i}$). With some mathematics the following simple relation can be derived from this system:

$$v_w = w + \sum_{i=1}^{N} c_j\, w_{s,j} \tag{11}$$

Substitution in (8) leads to the following result:

$$v_{z,i} = w + \sum_{i=1}^{N} c_j\, w_{s,j} - w_{s,i} \tag{12}$$

Apart from the vertical bulk velocity w, this result was already published by Smith (1966).

Numerical Procedure

When at a certain time the concentration of all fractions is known, the right-hand side of equation (12) is known and the grain velocity can be calculated. Using the grain velocities on time t the advection-diffusion equation can be solved for the next time step.

The partial differential equation (3) is solved for every fraction using a finite difference scheme. A semi-implicit approach is used. The first term in of the right-hand side (advection) is treated explicitly with a first order upwind scheme or with central differences; the second term (diffusion) is treated implicitly with central differences. This leads to a tri-diagonal system of equations.

The numerical procedure is as follows:
– Step 1. At the beginning of loading the concentration distribution is known for all fractions. In most cases $c_i = 0$ over total height (but can be arbitrarily chosen).
– Step 2. Using (12) the vertical velocities of the grains can be calculated.
– Step 3. Subsequently using the finite difference method all fractions are solved independently for one time-step. This leads to a new concentration distribution on time $t + \Delta t$.
– Step 4. Steps 2 and 3 are repeated until the hopper is filled to certain level or for a certain total simulation time.

The system can be solved when the following parameters and conditions are known:
– the initial condition (values of all quantities at $t = 0$);
– the boundary conditions;
– the value of the diffusion coefficient ε_z as a function of height z and time t;
– the value of the vertical bulk velocity w;
– the grain size distribution; and
– the value of $q_{s,i}$, the incoming sand flux per fraction.

A detailed description of the numerical procedure (Finite Difference Method) is beyond the scope of this article. Reference is made to Ferziger and Perić (1999). The boundary conditions and diffusion coefficient, however, deserve some extra attention.

Boundary Conditions

At the bottom the net sedimentation flux (depending on the concentration at the bottom) will be calculated, and this amount will be stored in the bed. At the water surface normally the sand flux will be put to zero. In this case at the surface the sand flux s will be prescribed to simulate the overflow:

$$s = v_{z,i} c_i \; (v_{z,i} > 0 \;\wedge\; Q_{in} > 0) \tag{13}$$

The two conditions must be included, since overflow will only take place when the mixture is discharged in the hopper, and to prevent the surface point from acting as a source term in case the vertical sand velocity is directed downwards.

Turbulent Diffusion Coefficient

It is common to relate the diffusion coefficient to the turbulent eddy viscosity using the Schmidt number σ_t:

$$\varepsilon_z = \frac{V_t}{\sigma_t} \tag{14}$$

Unlike the eddy viscosity, which is not a fluid property since it depends strongly on the flow field, the Schmidt number varies only slightly across any flow and also slightly from flow to flow (Rodi, 1993). Using the above relation, the problem has shifted towards the determination of the eddy viscosity. When focusing on the situation near the bottom in the density current, the order of magnitude of this parameter during the tests can be estimated using the mixing length theory of Prandtl:

$$\mathrm{v}_t = l_m^2 \left| \frac{\partial u}{\partial z} \right| \tag{15}$$

The mixing length increases with distance from the bottom. When δ is the thickness of the density current, the mixing length can be estimated as $0.09\,\delta$ (Rodi 1993). Typical values measured from the experiments are $\delta = 0.25$ m and $\partial u / \partial z = 2.5\ s^{-1}$. This leads to an eddy viscosity of 0.0013 m^2/s.

Comparison Between the 1DV Model and Experiments

The numerical model is compared with one-dimensional tests in a sedimentation column and the model hopper sedimentation tests.

Sedimentation column
The one-dimensional sedimentation tests were carried out in a tube with an inner diameter of 0.3 m and height of 1.5 m at the Laboratory of Fluid Mechanics at the Delft University of Technology (Runge et al. 1998 and Klerk and Meulepas 1998). Inside the tube a grid was placed, which was fixed to the walls of the column. By rotating the column (with grid), turbulence inside the column could be generated and the sedimentation of sand in turbulent conditions could be studied as well.

It is more common to generate turbulence by oscillating the grid into vertical direction inside a stationary column. In this case this approach could not

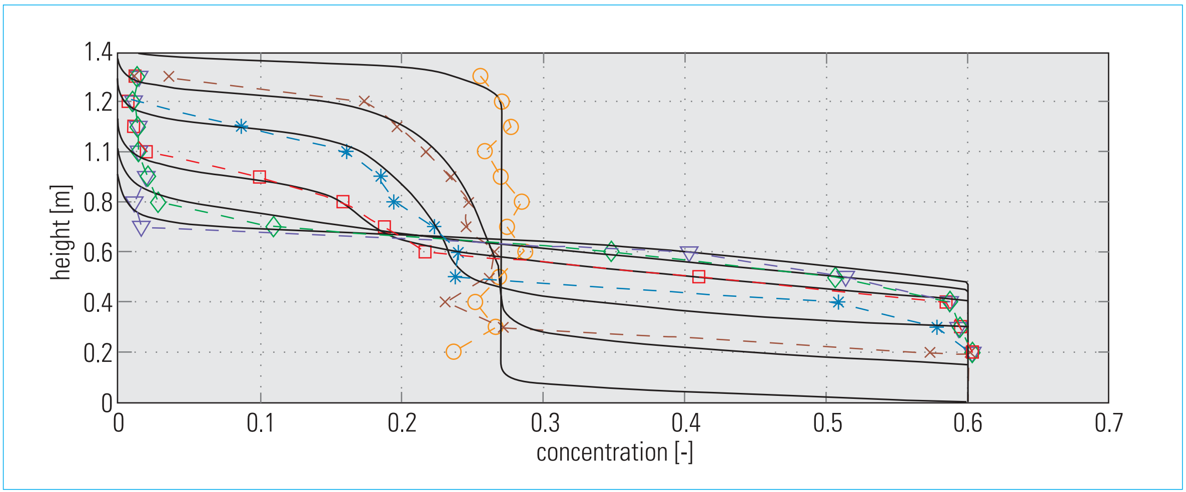


Figure 6. Calculated and measured concentration profiles at different times.

be used owing to the projected high initial sand concentration and the resulting high level of settled sediment inside the column after a test. The grid would in that case be forced to move through the sand bed that would lead to large forces on the grid and most probably destruction of the grid.

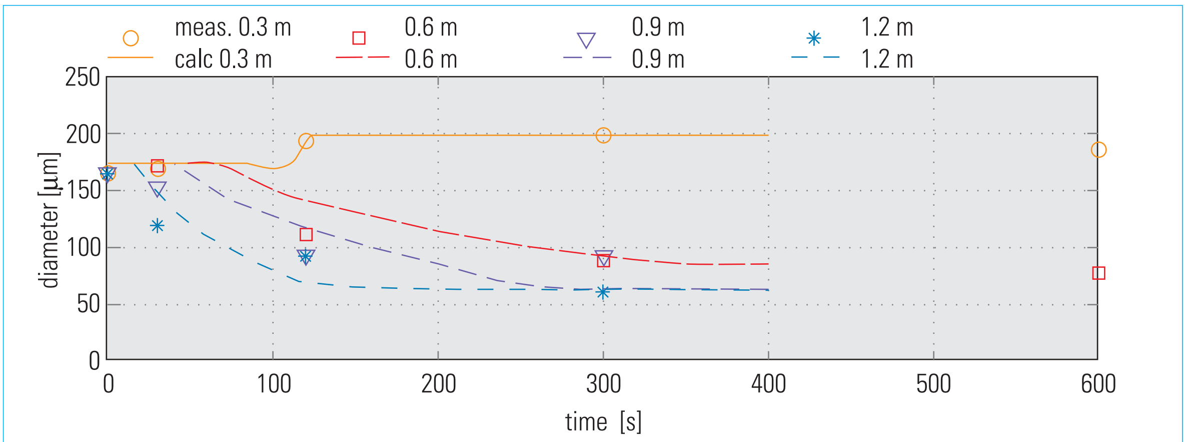
In the column, sand concentration was measured at twelve locations, so a good impression of the concentration vertical could be formed. Tests were carried out on uniform sands with grain diameter d_{50} of 80, 160 and 270 µm and graded sand with a d_{50} of 160 µm (d_{10} is 85 µm, d_{90} is 500 µm). Because the 1DV model includes the mutual interaction of the different fractions, results are shown here of the graded 160 micron tests. In Figure 6 results from a test are shown. The tube was filled to a level of 1.4 m with a sand concentration of approx. 30% by volume.

The measured and computed concentration profiles at different moments during the settling process are shown in Figure 6. The exponent in the hindered settlement function is according to the standard values of Richardson and Zaki (1954). The density correction according to Selim et al. (1983) was not used. The agreement between the measured and computed concentrations is good. During the test, samples were taken from the column at four different levels (0.3, 0.6, 0.9 and 1.2 m above the bottom of the column) at different times. The particle size distribution was determined from these samples.

During the calculation the concentration of all fractions is known as function of time and height. This enables calculation of the value of d_{50} in the model as function of time and space. The computed values can be compared with the measured values at the sampling locations in Figure 7.

The agreement is reasonable. The grain size in the settled bed is predicted quite well but the measured grain size in the suspension decreases faster than calculated from which can be concluded that the actual segregation develops more quickly than calculated.

Figure 7. d_{50} as function of time at different vertical positions.



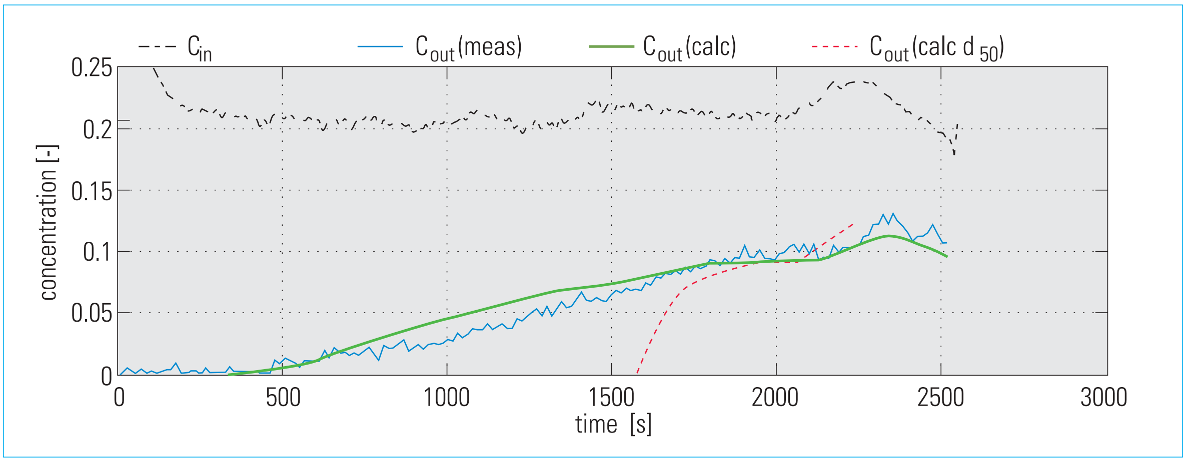


Figure 8. Comparison between measured and computed concentration during test 05.

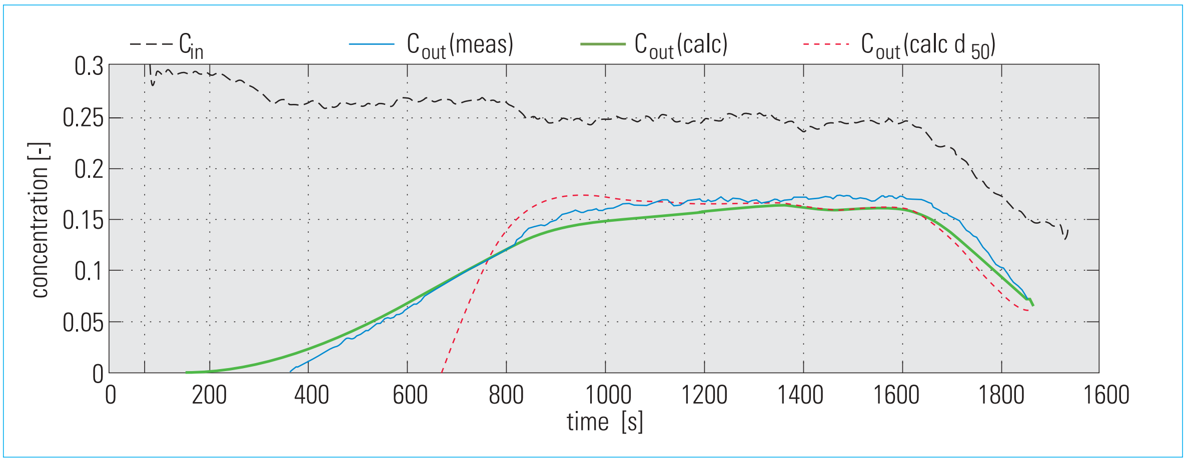


Figure 9. Comparison between measured and computed concentration during test 06.

Hopper sedimentation tests

Model hopper sedimentation tests were performed at WLIDelft Hydraulics (see Van Rhee 2001). The numerical model will be compared with two experiments Test 5 and Test 6. For both tests, sand with a d50 of 105 (m was used. The PSD is schematised to 7 fractions. Table II shows the values used in the calculation.

Operational parameters:

	Test 5	Test 6
Discharge	0.099	0.137 m^3/s
Density	1310	1420 kg/m^3
Overflow level	2.25	2.25 m
Water level at start	1.25	1.25 m

During the calculations the turbulent diffusion coefficient was constant over the height and equal to 0.0013 m^2/s. In Figures 8 and 9 the results of the comparison are shown. In these figures four lines are shown:

– the concentration of the mixture entering the hopper;
– the measured concentration in the overflow; and
– two computed concentrations in the overflow.

Table II. Values used in the calculation.

Diameter µm	Cumulative %
10	0
42	14.0
57	17.0
75	34.0
100	43.0
134	73.0
178	97.0
318	100.0

One overflow computation is based on a multi-sized mixture of the 7 fractions shown above and one calculation is based on a mono-sized mixture with a particle size equal to the d_{50} of the PSD. The results from the mono-sized mixture underestimate the overflow concentration during the largest part of the loading time (which leads to a lower cumulative overflow loss).

The final outflow concentration agrees, however, with the measurements. Test 6 was performed at maximum (for the installation) incoming sand flux. Owing to the high sand flux, the measured concentration in the overflow remains almost constant during some time. The model very well reproduces this phenomenon (which only occurs at very high or low sand flux). At the end of the test, the incoming sand flux decreases owing to a lack of sand in the storage tank (the overflow losses during the test were high, so a lot of sand was needed). The overflow concentration drops sharply in response. This phenomenon is reproduced nicely by the model.

It must be stressed, however, that it is not yet certain whether the model will reproduce the prototype situation as closely. On a larger scale, it is not known yet if the influence of the horizontal transport on the sedimentation velocity can be neglected (as is done in the 1DV model).

Conclusion

The flow field observed during the model hopper sedimentation tests formed a basis for the development of a relatively simple one-dimensional sedimentation model. This model is based on the advection-diffusion equation for a multi-sized mixture and includes the influence of the hopper load parameter and the particle size distribution.

The agreement between the numerical model and the experiments (one-dimensional sedimentation process in a sedimentation column and the model hopper sedimentation tests) is good. The model will now be extended to two dimensions to include the effect of the horizontal transport. Results of these efforts will be published in the future.

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