Alwin Albar

Effect of Various Terminal Velocity Equations on the Result of Friction Loss Calculation

Abstract

Several equations for calculating terminal velocity of sediment particles are presented. Each formula is then used to determine friction loss in slurry pipelines. The study evaluates the effect of each equation on the value of friction loss. Results are presented first as a function of slurry velocity, and then as a function of particle diameter. They are later compared with experimental data available in literature. It is found that there is a considerable difference between friction loss values yielded by those terminal velocity equations. This paper also discusses the advantages and disadvantages of applying these equations for friction loss calculation.

Introduction

Determining the terminal velocity is commonly used to calculate friction loss in a pipeline system. Previous researchers developed independently different methods to define the terminal velocity equation, mostly empirically. This resulted in various methods with their own characteristics and they claim their methods as satisfactory. The claims may be true for a particular case, however, it is not necessarily suitable for a particular friction loss calculation. This study will try to examine some methods that are commonly used for friction loss calculation in a pipeline.

In evaluating the various terminal velocity equations, this paper limits the discussion on Wilson's friction loss model only. The Wilson model (1992) has gained wide acceptance in the hydraulic transport industry. Matousek (1997) confirmed that the modelling approach by Wilson *et al.* (1992) recognises different slurry flow behaviour in flows with a different degree of flow stratification.

The Wilson friction loss model does not explicitly contain terminal velocity relations. The only parameter in the Wilson model that includes the terminal velocity is the particle-associated velocity (w). According to Wilson (1997), when the particle size becomes progressively finer, the value of the particle terminal velocity does not tend toward zero, as well as the value of w.

It would seem that the terminal velocity values would not influence the friction loss value very much, but it turns out that it does, considerably. Although there is no recommendation to use Wilson's method in calculating terminal velocity, it is possible that using it instead of others might be best.

Objective

The objective of this study is to observe the effect of the terminal velocity as evaluated from various terminal velocity equations on the result of Wilson friction loss calculations. Results of this observation will be used to help choose a suitable method of terminal velocity equation in friction loss calculation developed by Wilson *et al.* (1997), based on experimental data.

Terminal Velocity Equations

It is known that terminal velocity of the sediment particles plays a significant role in slurry transport. The most important factors generally considered are particle size, density and shape and ambient fluid properties. Various investigators have collected extensive data pertaining to terminal velocity of such particles and they have developed empirical equations to evaluate the terminal velocity. The following equations presented here were selected because of common use, simplicity and their recent development.

Equation developed by Schiller (1992)
Schiller (1992) developed an empirical relationship

Nomenclature

a	= maximum dimension of particle [m]
$A_{particle}$	= projected particle area for any particle [m^2]
A_{sphere}	= projected particle area for spherical particle [m^2]
Ar	= Archimedes number
b	= intermediate dimension of particle [m]
c	= minimum dimension of particle [m]
d_{50}	= median particle diameter [m]
d_{85}	= particle diameter for which 85% of the particles are finer [m]
d_n	= nominal particle diameter [m]
d_*	= dimensionless particle diameter (Cheng, 1997)
$d*$	= dimensionless particle diameter (Wilson, 1997)
f	= fluid-pipeline friction coefficient
g	= gravitational force [m/s^2]
M	= non-dimensional exponent coefficient
Re	= Reynolds number
SG_f	= specific gravity of fluid
SG_m	= specific gravity of mixture
SG_s	= specific gravity of particle
V_{sm}	= critical value of V_m at limit deposition
V_t	= particle terminal velocity [m/s]
V_{tf}	= dimensionless parameter
$V_{t(Sc)}$	= terminal velocity [mm/s]
w	= parameter of particle associated velocity
w_*	= non-dimensional fall velocity
β	= Corey shape factor
Δ	= $(\rho_s - \rho_f)/\rho_f$, dimensionless parameter
μ_f	= fluid dynamic viscosity [Ns/m^2]
μ_s	= mechanical friction coefficient of solids against pipewall
υ	= fluid kinematic viscosity [m^2/s]
υ_*	= non-dimensional kinematic viscosity
ρ_f	= fluid density [kg/m^3]
ρ_s	= sediment particle density [kg/m^3]
ψ	= particle sphericity, shape factor

using regression techniques based on data from Graf *et al.* (1966) and the result is:

$$V_{t(Sc)} = 134.14\left(d_{50} - 0.039\right)^{0.972} \qquad (1)$$

This equation is widely used because of its simplicity. It requires only the knowledge of the median grain size (d_{50}) in millimeters.

Equation developed by Swamee and Ojha (1991)

Swamee and Ojha derived the empirical equation for terminal velocity of non spherical particles based on the experimental data of Schulz *et al.* (1954). The proposed expression for terminal velocity is:

$$w_* = \left[\frac{44.84\upsilon_*^{0.667}}{\left(1+4.5\beta^{0.35}\right)^{0.833}} + \frac{0.794}{\left(\beta^4 + 20\beta^{20} + \upsilon_*^{2.4}\exp\left(18.6\beta^{0.4}\right)\right)^{0.125}}\right]^{-1} \qquad (2)$$

with non dimensional parameters:

$$w_* = \frac{V_t}{\sqrt{(SG_s - 1)gd_n}} \qquad (3)$$

$$\upsilon_* = \frac{\upsilon}{d_n\sqrt{(SG_s - 1)gd_n}} \qquad (4)$$

to remove the implicitness and avoid iteration process.

The nominal diameter (d_n) and Corey shape factor (β) were used in the formulation (Swamee and Ojha 1991). The nominal diameter is the diameter of a sphere of the same volume as the given particle, and defined as:

Table I. Correlation for dimensionless terminal velocity (V_{ts}*) as a function of dimensionless diameter (d*), after Grace (1986).

Range			Correlation
d*	V_{ts}*	Re_p	
≤ 3.8	≤ 0.624	≤ 2.37	$V_{ts}^* = (d^*)^2/18 - 3.1234\times10^{-4}\left(d^*\right)^5 + 1.6415\times10^{-6}(d^*)^8 - 7.278\times10^{-10}(d^*)^{11}$
3.8 to 7.58	0.624 to 1.63	2.37 to 12.4	$x = -1.5446 + 2.9162w - 1.0432w^2$
7.58 to 227	1.63 to 28	12.4 to 6370	$x = -1.64758 + 2.94786w - 1.09703w^2 + 0.17129w^3$
227 to 3500	28 to 93	6370 to 326000	$x = 5.1837 - 4.51034w - 1.687w^2 - 0.189135w^3$
			with: $w = \log_{10} d^*$; $x = \log_{10} V_{ts}^*$

$$d_n = \left(\frac{6V}{\pi}\right)^{1/3} \quad (5)$$

The Corey shape factor (β) is defined as:

$$ß = \frac{c}{\sqrt{ab}} \quad (6)$$

which was developed by considering the fact that the particles orienting themselves in the fluid presents the greatest resistance to the passing fluid. The Corey shape factor (β) is a logical dimensionless shape factor expressing the relative flatness of the particle where *ab* represents the particle projected area and *c* corresponds to the particle thickness.

Equation developed by Wilson et al. (1992)

Wilson (1992) calculates the settling velocity for a sphere falling in water and then corrects for the particle shape and subsequently considers hindered settling. For a sand particle that is assumed to be a sphere the terminal velocity is determined using the relationship tabulated in Table I. The following parameters (d^* and V_{tf}) are essential for calculating terminal velocity:

$$d^* = d_{50}\left[\frac{\rho_f(\rho_s - \rho_f)g}{\mu^2}\right]^{1/3} \quad (7)$$

$$V_{tf} = \left[\frac{\rho_f^{\,2}}{(\rho_s - \rho_f)\cdot g\cdot \mu}\right]^{1/3} \quad (8)$$

Once the value of V_{ts}^* is determined, the sphere terminal velocity V_{ts} can be calculated:

$$V_{ts} = \frac{V_{ts}^*}{V_{tf}} \quad (9)$$

In order to calculate non-spherical particle terminal velocity, the velocity ratio ξ must be obtained from the following chart after determining the correct volumetric shape factor k from Table II :

The actual terminal velocity can then be obtained by correcting V_{ts} using the following equation.

$$V_t = \xi \cdot V_{ts} \quad (10)$$

Equation developed by Hartman et al. (1994)
Hartman *et al.* (1994) conducted experiments on limestone and combined their results with data of Pettyjohn and Christiansen (1948). The materials were selected so that the particles had approximately equal axes at right angle to each other. More than 400 experimental data points were fitted by minimising the standard deviation between the experimental values and the values estimated from the proposed relationship.

Alwin Albar (center) recipient of the IADC Award is seen here with Mr Peter Hamburger (left), Secretary General of the IADC, and Dr Robert Randall, Director of the Center for Dredging Studies at Texas A&M University.

IADC Award 2000

Presented at the WEDA XX and 32nd Texas A&M University Dredging Seminar, Warwick, Rhode Island, USA June 25-28 2000

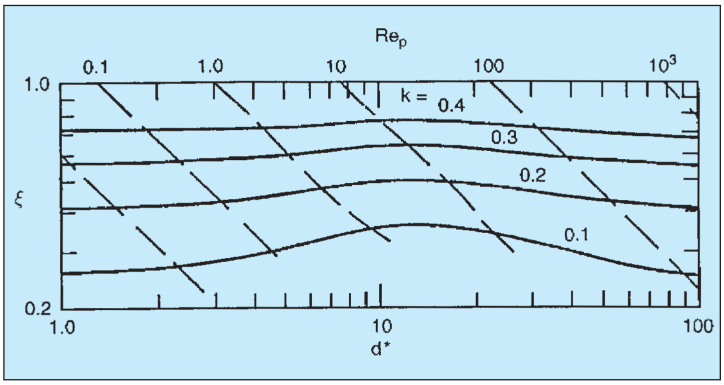
At the Twentieth Western Dredging Association Annual Meeting and Conference, held in late June this year, Alwin Albar was presented with the annual IADC Award for young authors. Mr Albar received his BSc in Mechanical Engineering from Bandung Institute of Technology, Indonesia, in October 1992. He proceeded with his study, earning a MSc in Mechanical Engineering from the University of Wisconsin at Madison in May 1995. In 1996 he was recruited by PT Timah Tbk, an Indonesian tin-mining company, and then received a full scholarship to pursue a PhD in Ocean Engineering at Texas A&M University, College Station, Texas where he is presently enrolled in the Ocean Engineering Programme of the Civil Engineering Department.

Each year at a selected conference, the International Association of Dredging Companies grants an award to a paper written by an author younger than 35 years of age. The Paper Committee of the conference is asked to recommend a prize-winner whose paper makes a significant contribution to the literature on dredging and related fields. The purpose of the award is "to stimulate the promotion of new ideas and encourage younger men and women in the dredging industry". The IADC Award consists of US$ 1000, a certificate of recognition and publication in *Terra et Aqua*.

Table II. Volumetric shape factors for isometric and mineral particles (Wilson 1997).

Isometric Particles	k	Mineral Particles	k
Sphere	0.524	Sand	0.26
Cube	0.696	Silliminate	0.23
Tetrahedron	0.328	Bituminuos Coal	0.23
Rounded	0.54	Blast furnace slag	0.19
Sub-angular (partly rounded)	0.51	Limestone, talc, plumbago	0.16
Sub-angular (tending to caboodle)	0.47	Gypsum	0.13
Sub-angular (tending to tetrahedral)	0.38	Flake graphite	0.023
		Mica	0.003

Figure 1. Ratio of the terminal velocity of a non-spherical particle to the value for a spherical particle, ξ, as a function of dimensionless diameter, d (Wilson 1997).*



Hartman used the sieve diameter and sphericity shape factor (ψ) in the calculation (Hartman et al. 1994). The sphericity shape factor (ψ) is defined as:

$$\psi = \frac{A_{sphere}}{A_{particle}} \qquad (11)$$

The more ratio departs from unity, the lower the value of sphericity (ψ = 1 corresponds to a sphere). It is difficult to determine ψ directly in the case of irregular particles.

The proposed relationship for Reynolds number for any given particle dimension that implicitly include the terminal velocity (V_t) is:

$$\log \mathrm{Re}(A_r,\psi) = \log \mathrm{Re}(A_r,1) + P(A_r,\psi) \qquad (12)$$

where:

$$V_t = \frac{\nu \cdot d_{50}}{\mathrm{Re}} \qquad (13)$$

$$\log \mathrm{Re}(A_r,1) = -1.2738 + 1.04185 \log A_r$$
$$- 0.060409(\log A_r)^2 + 0.0020226(\log A_r)^3 \qquad (14)$$

and:

$$P(A_r,\psi) = -0.071876(1-\psi)\log A_r$$
$$- 0.023093(1-\psi)(\log A_r)^2 + 0.0011615(1-\psi)(\log A_r)^3$$
$$+ 0.075772(1-\psi)(\log A_r)^4 \qquad (15)$$

$$A_r = d_{50}{}^3 g \frac{\rho_f(\rho_s - \rho_f)}{\mu_f{}^2} \qquad (16)$$

Equation developed by Cheng (1997)

Cheng (1997) proposed a recent empirical relationship for terminal velocity of non spherical particles.
A simplified explicit formula was evaluated based on experimental data of Schiller and Naumann (1933) and US Inter Agency Committee (1957). Cheng limited his formulation to natural sediment only and did not clearly stated the dimension definition used in the calculation, although the paper implicitly stated that the shape factor used is the Corey shape factor (β=0.7) and the diameter used in the calculation is the sieve diameter (Cheng 1997). The proposed formula for terminal velocity is:

$$\frac{V_t \cdot d_{50}}{\upsilon} = \left(\sqrt{25 + 1.2 d_*{}^2} - 5\right)^{1.5} \qquad (17)$$

with:

$$\Delta = \frac{(\rho_s - \rho_f)}{\rho_f} \qquad (18)$$

$$d_* = d_{50} \cdot \left(\frac{\Delta g}{\upsilon^2}\right)^{1/3} \qquad (19)$$

Friction Loss Calculation

There are many approaches to calculate slurry flow friction loss such as empirical, microscopic and macroscopic approaches. The first predictive tools were empirical approaches developed in the 1950s, which predicted basic slurry pipeline characteristics. The next was a microscopic approach, which defined the laws governing slurry flow for an infinitesimal control volume of slurry, developed in the 1980s. A macroscopic approach offers good compromise between the other two. This approach applies the balance (conservation) equation to a larger control volume of slurry.
An example of such a control volume is a unit length of pipe length containing an approximately uniform concentration of solids.

Wilson (1970) introduced the concept based on the principle of force balances in the two-layer pattern of mixture flow stratified into a bed load and a suspended load. Furthermore, Wilson (1992) developed a new semi-empirical model for heterogeneous flow in slurry pipelines, which was calibrated by using experimental data. This model is based on considering heterogeneous flow as a transition between two extreme flows governed by a different mechanism for support of a solid particle in the stream of the carrier liquid which are fully stratified flow and fully suspended flow. Resistance in fully stratified flow is predominantly a result of mechanical friction between solid particles and the pipeline wall. The frictional head loss is predicted by using a two-layer model.

Wilson *et al*. (1992) introduced the parameter V_{50} that expresses the mean slurry velocity at which one half of the transported solid particles contribute to a suspended load and half by contact with other particles.
This equation expresses the influence of suspension mechanisms for the carrier fluid turbulent diffusion and the hydrodynamic lift acting on particles larger than the sub-layer thickness in the near wall region.
The V_{50} can be estimated by equation:

$$V_{50} = w\sqrt{\frac{8}{f}}\cosh\left(\frac{60 \cdot d}{D}\right) \tag{20}$$

where:

$$w = 0.9 v_t + 2.7\left[\frac{(\rho_s - \rho_f) g\mu}{\rho_f^{\,2}}\right]^{1/3} \tag{21}$$

The relationship between the relative solids effect and mean slurry velocity is given as:

$$\frac{I_m - I_f}{SG_m - 1} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M} \tag{22}$$

The exponent M is given by the expression

$$M = \left[\ln\left(\frac{d_{85}}{d_{50}}\right)\right]^{-1} \tag{23}$$

The value of M should not exceed 1.7, the value of narrow-graded solids, or fall below 0.25. In practice the value of M is considered to be equal to 1.7.

The Wilson model gives a scale-up relationship for friction loss in slurry pipelines of different sizes transporting solids of different sizes at different concentrations. It is based on the assumption that there is a power relationship between the relative solid effect and the mean slurry velocity that is valid in all slurry flow conditions. The exponent M of this relationship is assumed to be dependent on the particle size distribution only.

Wilson (1992) also proposed a particular relation for calculating the deposition-limit velocity (V_{sm}), which is a minimum value of flow velocity to enable sediment to be transported. The relation is:

$$V_{sm} = \frac{8.8\left(\frac{\mu_s (SG_s - SG_f)}{0.66}\right)^{0.55} D^{0.7} d_{50}^{\,1.75}}{d_{50}^{\,2} + 0.11 D^{0.7}} \tag{24}$$

Case Study

The objective of the first case study is to compare the friction loss values using all terminal velocity models and experimental data (Potnis 1997). Each model is plotted as a function of flow velocity for a particular pipe size, specific gravity and particle size. To make them comparable, values used in the calculation are the same with values used in the experiment.
Potnis (1997) measured the experimental data of sand (SG_s = 2.65) for 10.16 cm (4 in.), 15.24 cm (6 in.) and 20.32 cm (8 in.) pipes with particle diameter 0.3 mm.

It can be seen from the graphs (Figure 2) of the first case study that the Wilson method gives the closest result to the experimental data. Based on this fact, the next case study is meant to compare other models with the Wilson method. In other words, the Wilson method is used as a reference for others because of its best results. With this in mind, graphs in this next case study (Figures 3 and 4) showing differences with the Wilson model, are expected to give an overview of the other models performance. The performance is expressed by percent difference (results of each model subtracted by the results of reference model, divided by the results of reference model).

The graphs shown here are samples for a particular

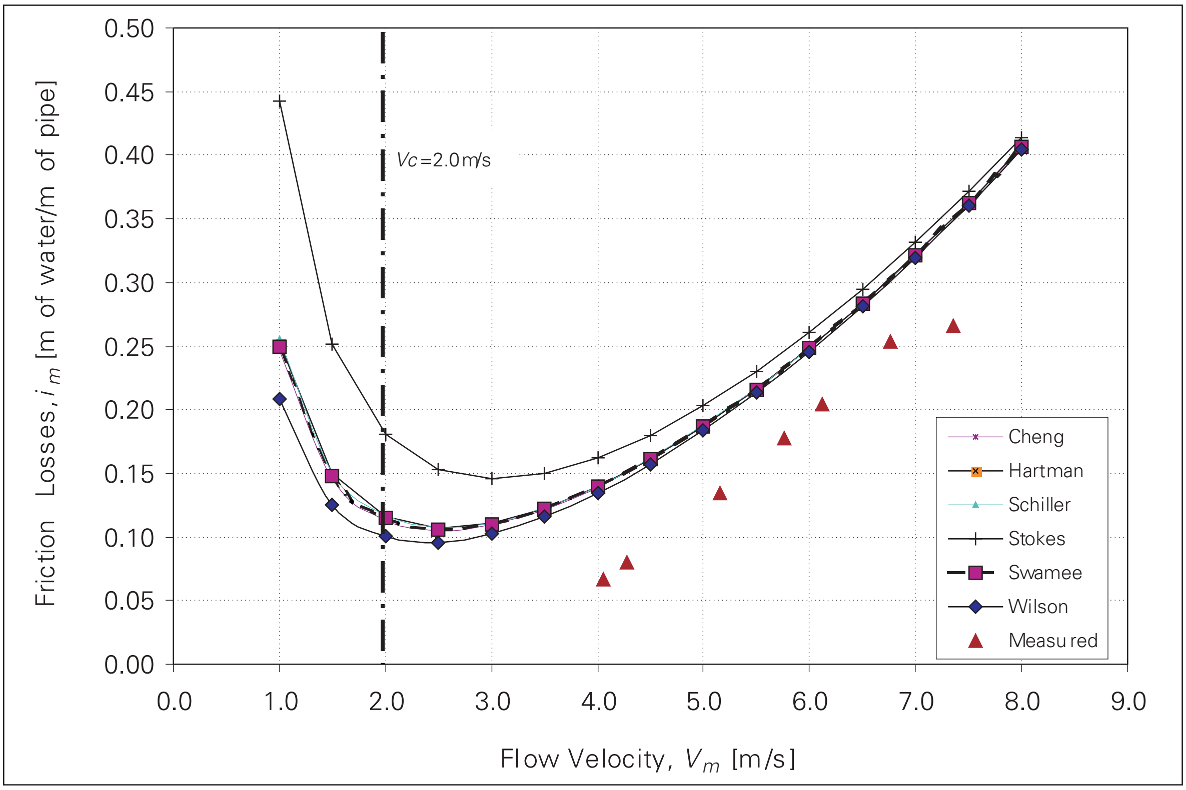


Figure 2. Variation of friction loss with flow velocity, pipe size 4 in., particle size = 0.3 mm.

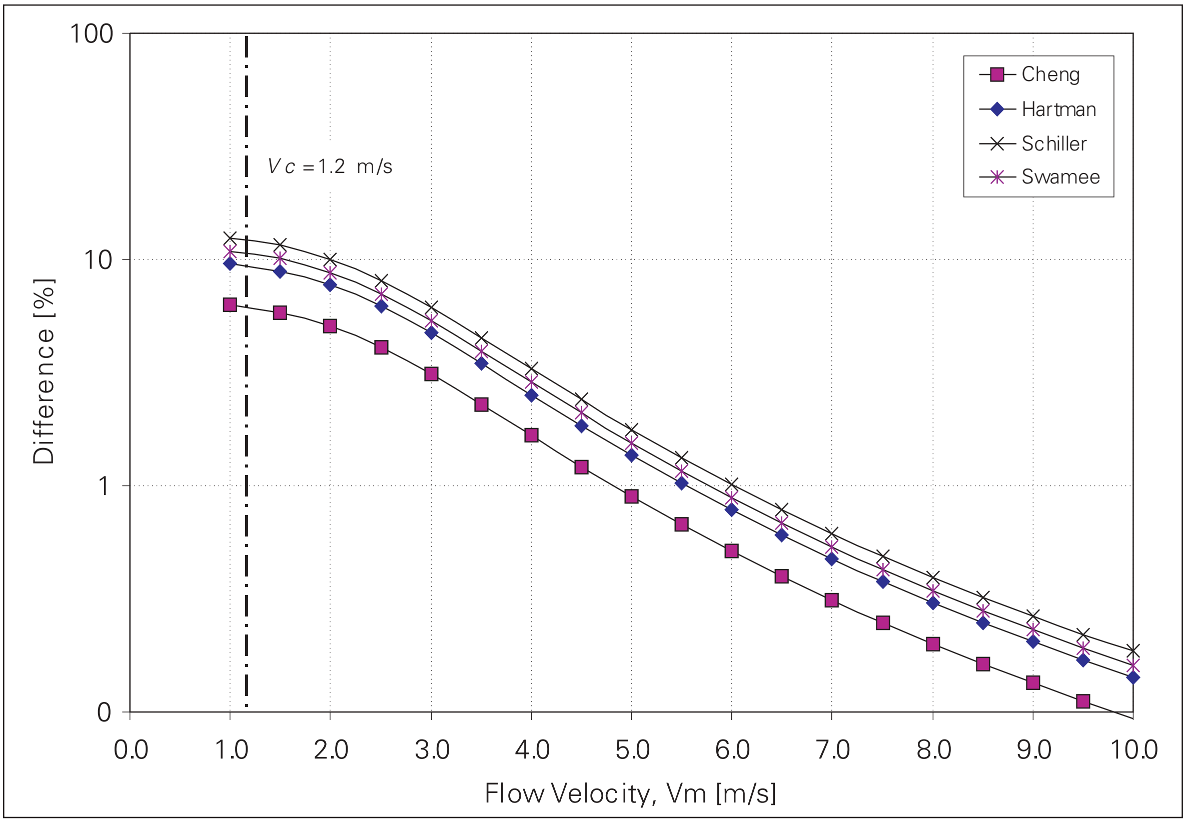


Figure 3. Percent difference of friction loss compared to Wilson model pipe size D=8 in., particle size d = 0.1 mm.

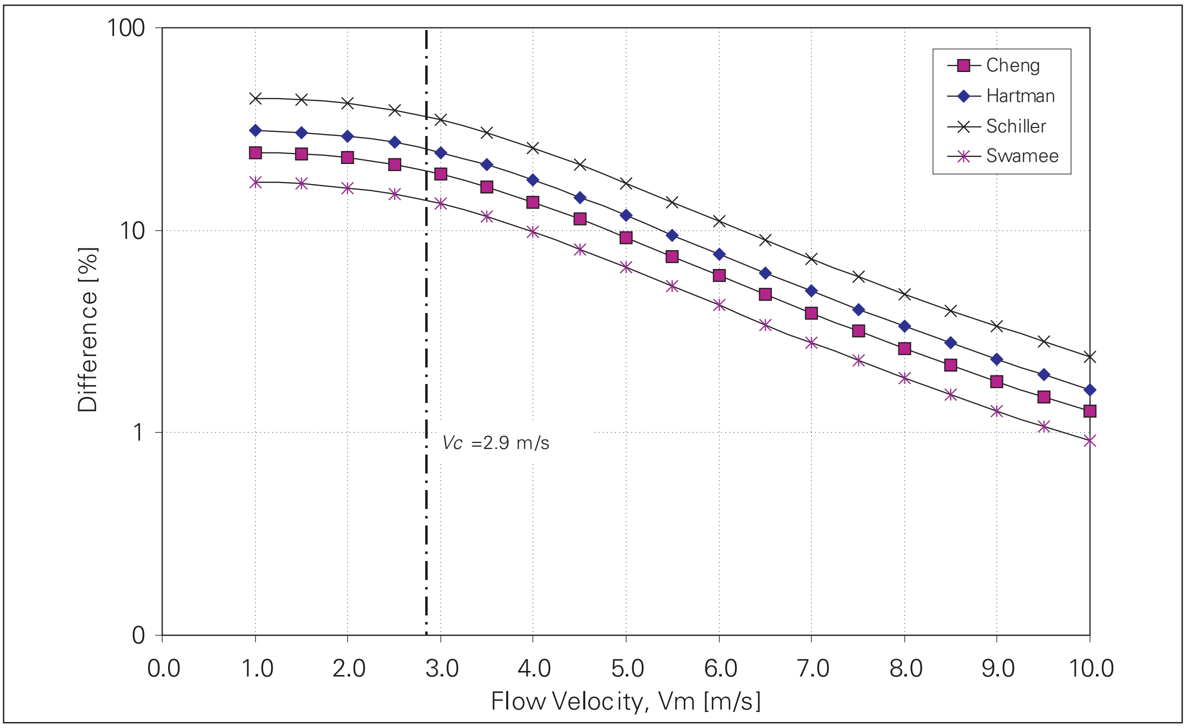


Figure 4. Percent difference of friction loss compared to Wilson model pipe size D=8 in., particle size d = 1.0 mm.

pipe size. Results for other pipe sizes, although not shown here, exhibit similar behaviour. Figures from the second case study show that each model gives different performance as the particle size changes. This leads to the last case study, where the behaviour of each model as a function of parameters is studied extensively by varying simultaneously the particle size and flow velocity. Figure 5 shows the behavior of the Wilson model itself. The other figures (Figures 6 through 9) present the percent difference of the other models with the plot of Figure 5.

Discussion

Five equations for evaluating the particle settling (terminal) velocity have been reviewed and the following observations about the different methods are made:

- The Swamee equation (Eq. 3) is valid for any range of particle grain size and for any specific gravity. Although not very simple, it is relatively easy to use in the sense that no iteration is needed.
- The Schiller model is a very simple equation, however its simple form results in some limitations. The grain size is limited to 2 mm maximum, and valid only for materials that have a specific gravity equal to sand.
- The Wilson equation is the most complicated of all the equations presented here. It is valid for any range of grain size and for any specific gravity. The complexity is increased by certain conditions for different ranges of dimensionless particle diameters, which make it difficult to calculate. Moreover, the particle shape factor used in the Wilson equation is not commonly used.
- The Hartman particle shape factor is not easy to calculate. Although not as complex as the Wilson equation, it is still a long and complex equation. One advantage is that it is valid for any range of grain size and for any specific gravity.
- The Cheng equation is the simplest and is limited to natural sand only. It is valid for only one particle shape factor.

The Wilson friction loss calculation is a function of many parameters. Amongst these parameters are parameters affecting both terminal velocity and the friction loss directly, for example particle diameter (*d*) and particle specific gravity (SG_S). On the other hand, there are parameters affecting the values of friction loss only, such as pipe diameter (*D*) and flow velocity (V_m). However, there is also the particle shape factor (*PSF*), which affects terminal velocity only.

To show how the terminal velocity affects the friction loss values, this study chooses particle diameter as the varied parameter owing to the consideration that it gives the greatest influence on the values of terminal velocity. At this time, the study is limited for a single material, which has certain values of particle shape factor (*PSF*) and SG_S. Other parameters varied to observe their influence on the friction loss are *D* and V_m.

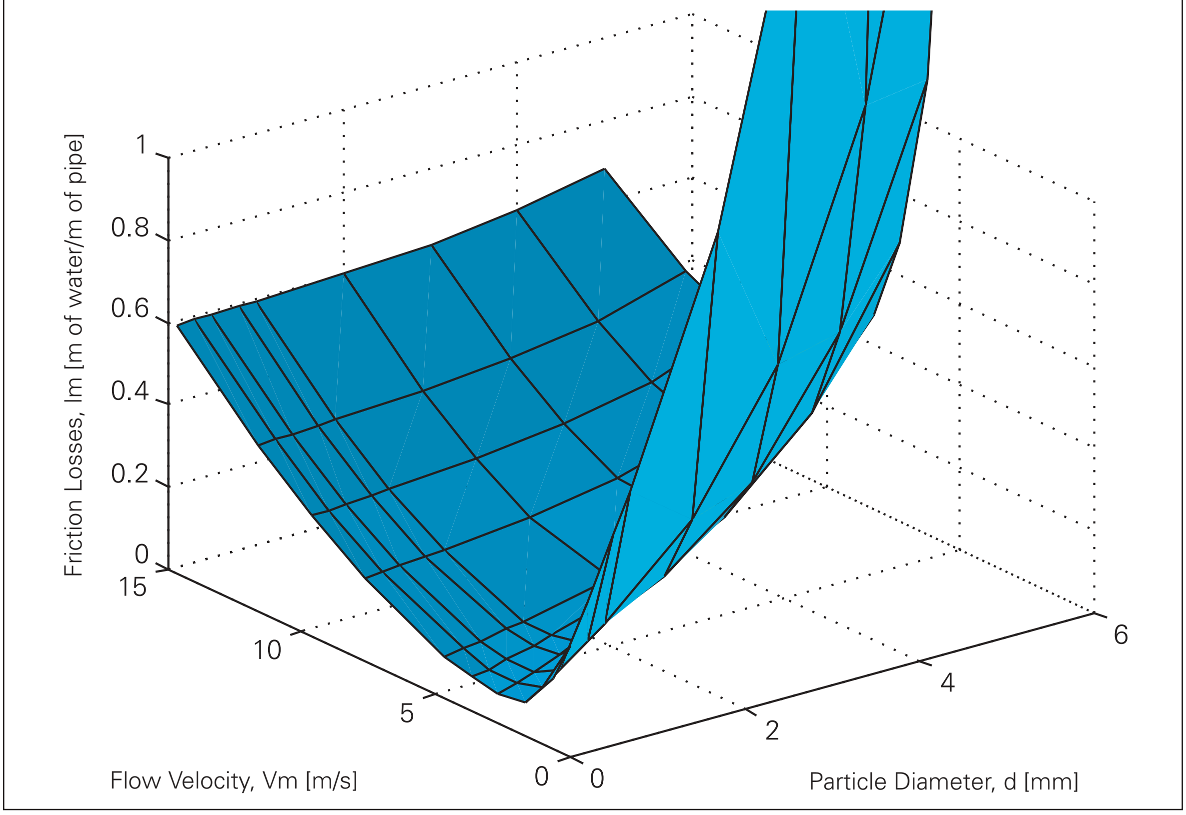


Figure 5. Variation of friction loss with various flow velocity and particle diameter pipe size D = 8 in.

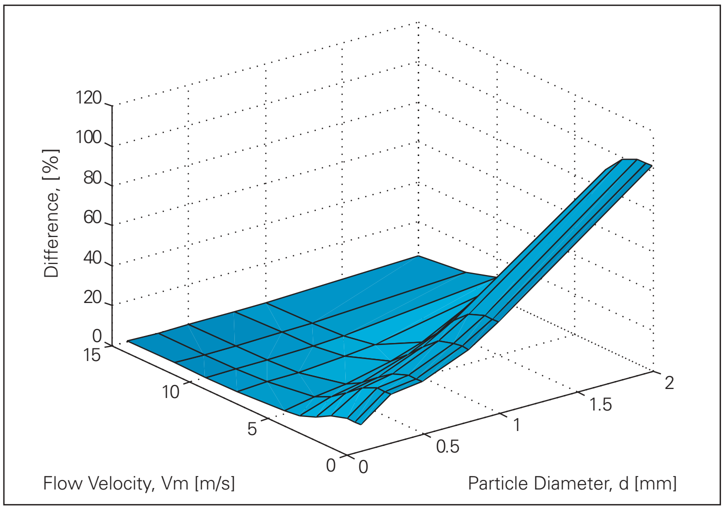


Figure 6. Percent difference of friction losses using Cheng equation compared to Wilson model pipe size D = 8 in.

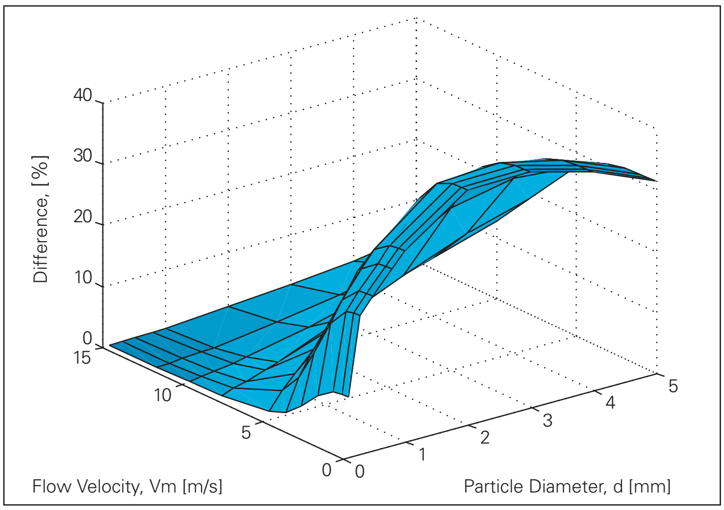


Figure 7. Percent difference of friction losses using Hartman equation compared to Wilson model pipe size D = 8 in.

The first case study (Figure 2) shows that every equation presented in this study exhibits a similar trend in calculating the friction loss as a function of V_m.

As mentioned before, the Wilson method is used as a reference because it has the closest results to all experimental data. This study shows that pipe size affects only the values of friction loss but not the trend of each model. This makes the graphs shown here reflect the trend (not value) of friction loss for all pipe sizes.

Results from the second and third case studies show that other equations always have a positive difference compared to Wilson method. These results also show that each terminal velocity equation has its own characteristic. The following is a summary of these characteristics for an 8-inch pipe diameter:

– The Swamee equation gives consistent results in all ranges of particle size and flow velocity (Figure 9), meaning that the percent difference is almost constant for those ranges. The difference peaks at 21% at V_m= 0.1 m/s and d = 0.5 mm. An interesting characteristic of this equation is that the peak difference occurs at particle sizes below 1 mm.
– At smaller ranges of particle size, the Schiller equation agrees well (Figure 8). However, as grain size increases, the percent difference rises drastically. The peak difference is the highest amongst the models presented in this paper (102% at V_m=0.1 m/s and d= 2 mm).
– The Hartman equation gives the maximum difference 38% at V_m=0.1 m/s and d= 3 mm (Figure 7). Similar to the Swamee equation, the peak difference does not occur at large particle sizes.
– Amongst models, Cheng gives best results for particle sizes up to 0.5 mm. However, the

difference grows rapidly as particle size increases (Figure 6). Maximum difference = 32% is found at the highest particle size from the data set (d = 5 mm) and V_m = 0.1 m/s.

Overall, the most consistent results are given by the Swamee equation. For smaller particle sizes the Cheng equation gives the best results amongst others. Its performance degrades as particle size increases. For example, above d = 0.5 mm, the Swamee equation outperforms the Cheng equation in terms of percent difference. The other two equations always have larger differences than the Swamee and Cheng equations.

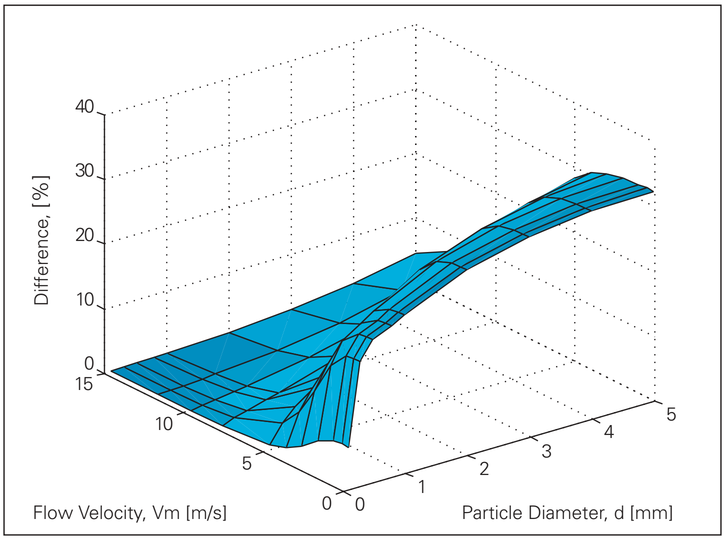


Figure 8. Percent difference of friction losses using Schiller equation compared to Wilson model pipe size D = 8 in.

Conclusions

This study addresses the issue of the friction loss calculation affected by various terminal velocity equations. It is observed that the largest influence of terminal velocity is in low flow velocity and coarser particle grain size. The first case study showed that using the Wilson method in calculating terminal velocity gives the closest results to measurements. The second and third case studies gave an overview of the characteristics of each equation on friction loss calculation. It is hoped that these results can give some insight on the differences of incorporating various terminal velocity equations into the friction loss calculation.

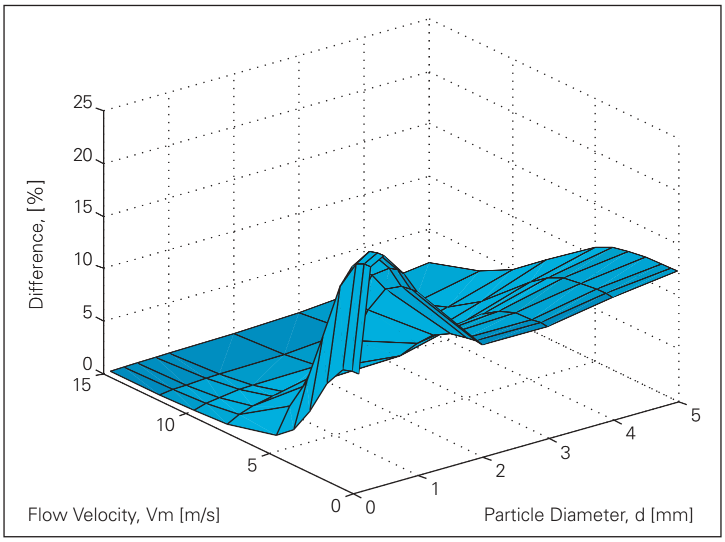


Figure 9. Percent difference of friction losses using Swamee equation compared to Wilson model pipe size D = 8 in.

References

Cheng, N. S. (1997).
"Simplified settling velocity formula for sediment particle."
J. Hydr. Engr. Div., ASCE, 123 (2), pp 149-152.

Hartman, M., Trnka, O., and Svoboda, K. (1994).
"Free settling of non-spherical particles."
Ind. Eng. Chem. Res., 33 (8), pp 1979-1983.

Herbich, J. B. (1992).
Handbook of Dredging Engineering.
McGraw Hill Publishers, New York, USA.

Matousek, V. (1997).
Flow Mechanism of Sand-Water Mixtures in Pipelines,
Ph.D. Dissertation, Delft University of Technology,
The Netherlands.

Potnis, S. (1997).
Automated Data Acquisition of Experimental Head Loss in Slurry-carrying Pipes.
M.E. Major Report, Texas A&M University, College Station, TX, USA.

Randall, R. E. (1999).
Class Notes, Sediment Transport in Pipes, OCEN-688, Marine Dredging Course.
Texas A&M University, College Station, TX, USA.

Swamee, P. K. and Ojha, C. S. (1991).
"Drag coefficient and fall velocity of non-spherical particles."
J. Hydr. Engr. Div., ASCE, 117 (5), pp 660-667.

Wilson, K. C., Addie, G. R. and Clift, R. (1997).
Slurry Transport Using Centrifugal Pumps.
Elsevier Applied Science Publishers, New York, USA.